Methuen's Monographs on Physical Subjects

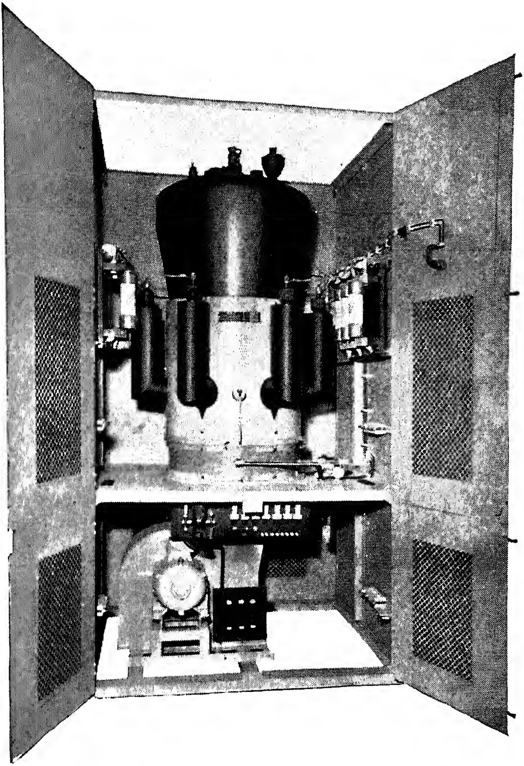
General Editor: B. L. Worsnop, B.Sc., Ph.D.

MERCURY ARCS

METHUEN'S MONOGRAPHS ON PHYSICAL SUBJECTS

General Editor: B. L. WORSNOP, B.Sc., Ph.D.

APPLETON, E. V.	THERMIONIC VACUUM TUBES
ARNOT, F. L.	COLLISION PROCESSES IN GASES
BEECHING, R.	ELECTRON DIFFRACTION
BRAILSFORD, F.	MAGNETIC MATERIALS
CHAPMAN, S.	THE EARTH'S MAGNETISM
CHILDS, W. H. J.	PHYSICAL CONSTANTS
DINGLE, H.	THE SPECIAL THEORY OF RELATIVITY
EMELÉUS, K. G.	THE CONDUCTION OF ELECTRICITY THROUGH GASES
FEVRE, R. J. W. LE	DIPOLE MOMENTS
FLINT, H. T.	WAVE MECHANICS
FRANCIS, V. J.	FUNDAMENTALS OF DISCHARGE TUBE CIRCUITS
FRASER, R. G. J.	MOLECULAR BEAMS
HAGUE, B.	AN INTRODUCTION TO VECTOR ANALYSIS FOR PHYSICISTS AND ENGINEERS
HIRSCHLAFF, E.	FLUORESCENCE AND PHOSPHORESCENCE
JACKSON, L. C.	LOW TEMPERATURE PHYSICS
JACKSON, L. C.	WAVE FILTERS
JACKSON, WILLIS	HIGH FREQUENCY TRANSMISSION LINES
JACOB, L.	HIGH VOLTAGE PHYSICS
JAMES, R. W.	X-RAY CRYSTALLOGRAPHY
JOHNSON, R. C.	ATOMIC SPECTRA
JONES, T. J.	THERMIONIC EMISSION
JOSEPHS, H. J.	HEAVISIDE'S ELECTRIC CIRCUIT THEORY
KNUDSEN, MARTIN	THE KINETIC THEORY OF GASES
LAMONT, H. R. L.	WAVE GUIDES
McCREA, W. H.	RELATIVITY PHYSICS
McVITTIE, G. C.	COSMOLOGICAL THEORY
MANN, W. B.	THE CYCLOTRON
OATLEY, C. W.	WIRELESS RECEIVERS
OWEN, D.	ALTERNATING CURRENT MEASUREMENTS
PORTER, A. W.	THE METHOD OF DIMENSIONS
PORTER, A. W.	THERMODYNAMICS
RATCLIFFE, J. A.	THE PHYSICAL PRINCIPLES OF WIRELESS
SAS, R. K. and PIDDUCK, F. B.	THE M.K.S. SYSTEM OF ELECTRICAL UNITS
SCHONLAND, B. F. J.	ATMOSPHERIC ELECTRICITY
SOMMER, A.	PHOTOELECTRIC CELLS
STONER, E. C.	MAGNETISM
STYLE, D. W. G.	PHOTOCHEMISTRY
SUTHERLAND, G. B. B. M.	INFRA-RED AND RAMAN SPECTRA
TEAGO, F. J., and GILL, J. F.	MERCURY ARCS
TEAGO, F. J.	THE COMMUTATOR MOTOR
TEMPLE, G.	THE GENERAL PRINCIPLES OF QUANTUM THEORY
TOLANSKY, S.	HYPERFINE STRUCTURE IN LINE SPECTRA AND NUCLEAR SPIN
WHITE, F. W. G.	ELECTROMAGNETIC WAVES
WILLIAMS, W. E.	APPLICATIONS OF INTERFEROMETRY
WILSON, A. J. C.	X-RAY OPTICS
WORSNOP, B. L., and CHALKLIN, F. C.	X-RAYS



Steel Pumpless Rectifier (500 kW at 440 Volts)

MERCURY ARCS

by

F. J. TEAGO

D.Sc., M.I.E.E.

ROBERT RANKIN PROFESSOR OF ELECTRICAL ENGINEERING
ELECTROTECHNICS IN THE UNIVERSITY OF LIVERPOOL
CHARTERED ELECTRICAL ENGINEER

and

J. F. GILL

M.Sc., A.M.I.Mech.E., A.M.I.E.E.

WITH A FRONTISPIECE
AND 50 DIAGRAMS

METHUEN & CO. LTD., LONDON

36 Essex Street, Strand, W.C.2

First published September 10th 1936
Second edition, revised, 1949

CATALOGUE NO. 4021/U

PRINTED IN GREAT BRITAIN

PREFACE

THIS monograph has formed the basis of a course of Design Lectures on the Mercury Arc given, during recent years, in the Department of Electrical Engineering in the University of Liverpool, and briefly outlines the more important aspects of the arc from the electrical, mechanical, and operational standpoints.

It is not intended to replace the much more detailed works referred to in the Bibliography; in fact, fine detail has purposely been omitted in order that the essential principles may stand out clearly.

Detail often obscures a preliminary study and renders it difficult, but once the main principles are recognized and understood one may, as far as is profitable for the purpose in view, add detail.

The author wishes to thank the General Electric Co., Ltd., England, for permission to publish the constructional details given in the text and the photograph which forms the frontispiece.

To me has fallen the task of 'bringing the material into due form' by reason of the lamented death of my great friend Mr. Gill, in November 1935. 'He gave his help in full measure and without stint.'

F. J. TEAGO

LIVERPOOL UNIVERSITY
January 1936

PREFACE TO SECOND EDITION

THE monograph has been revised and the text and figures have undergone changes where experience has shown such changes to be desirable.

Chapter X has required most alteration, which consists principally in redrawing figs. 42 and 44 in order that these hypothetical wave shapes should conform more closely to actual oscillograph records. These alterations do not, however, necessitate alterations to the text involved. Of the additions made the most substantial is an outline of the theory of the unbalanced Post Office Box in order to give a better understanding of the working of the Pirani Gauge.

LIVERPOOL UNIVERSITY
September 1947

CONTENTS

BIBLIOGRAPHY

C. P. STEINMETZ: *Alternating Current Phenomena*, McGraw-Hill

J. AMBROSE FLEMING: *Mercury Arc Rectifiers and Lamps*, Pitman & Sons

L. B. W. JOLLEY: *Alternating Current Rectification*, Chapman & Hall

H. RISSIK: *Mercury Arc Current Convertors*, Pitman & Sons

C. R. BURCH and C. SYKES: 'Continuously Evacuated Valves', *Journal I.E.E.*, July 1935

R. J. CLARK: 'The McLeod Gauge', *Journal Scientific Insts.*, Vol. V, 1928

F. J. TEAGO: *The Multiphase Mutator*

'No-Load E.M.F. Wave Shape with Retarded Commutation', *The Electrician*, Vol. CXX, No. 3122, 1938

CHAPTER I

INTRODUCTION

STATIC means for converting alternating to direct current depend for their action on the unilateral conductivity of various substances, solid, liquid, and gaseous, and each type has its own peculiar characteristics which render it useful for certain duties.

Such applications as crystal detectors and diode valves for wireless receivers, copper-oxide and selenium rectifiers for handling larger outputs and whose applications to industry are rapidly extending, are typical examples of the use of solids.

Liquid rectifiers having one electrode of lead and the other of tantalum, in a solution of ammonium phosphate, pass current through the cell from lead to tantalum and have been used for battery charging on a relatively small scale.

The most successful rectifier for moderate and large powers is the type depending on the unilateral conductivity of an ionized vapour, mercury vapour being that in general use, because of its particularly suitable properties of which the chief one is that a mercury cathode in conjunction with a graphite anode gives complete rectification, whereas most other types give partial rectification only.

The rectifying properties of mercury vapour were first utilized by Cooper-Hewitt in 1903 and since that date the development of the mercury vapour rectifier has been very rapid. For the degree of vacuum required and the comparatively large size of the containers involved, a practically new technique had to be developed and with the advent of the metal container further researches were needed to enable vacuum-tight seals and satisfactory welded joints to be obtained. The introduction of grid control has contributed largely to the success of the mercury vapour rectifier and has opened up prospects of still further development.

The simplest practical form of mercury arc rectifier in common use comprises an evacuated vessel containing a cathode in the form of a pool of mercury and two anodes, usually of graphite or carbon, arranged symmetrically with respect to the cathode and effectively shielded to prevent the direct impact of molecules of mercury vapour which may be projected from the cathode. Where it is desired to make use of the valuable properties of grid control, the rectifier is also provided with an insulated grid surrounding each anode.

The containing vessel for the smaller rectifiers is of glass through which the various electrodes are sealed and it is then charged with mercury and exhausted to a pressure of about 0·0005 mm. of mercury at normal atmospheric temperature to remove all traces of air and other injurious gases.

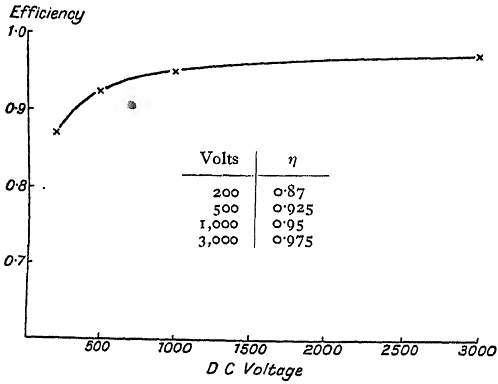
This form of apparatus is quite satisfactory for outputs of 500 amperes at voltages not exceeding 3,000 as it is usually well protected against mechanical or electrical damage, and since, due to the vacuum, there is no consumption of material nor destructive sparking, the maintenance cost is nil and the maximum output is limited only by the temperature rise of the glass container which may be cooled by means of a fan. For larger outputs at voltages above 3,000 it is desirable to provide a water-cooled metal container, and in this case, chiefly owing to the difficulty of maintaining the joints absolutely tight, it is made demountable and is provided with means for continuously evacuating it.

Up to the present time voltages as high as 60,000 and currents up to 10,000 amperes have been obtained and rectifiers rated at 30 amperes, 20,000 volts and 5,260 amperes, 575 volts have been put into service and many thousands of kW of rectifiers with control grids are now being manufactured for various purposes.

The efficiency of the rectifier depends upon the voltage; at 1,000 volts it is more efficient than 50 cycle rotary converting plant, and at 2,000 volts and upwards the superiority is very marked. The arc drop is not very dependent upon the current so that the efficiency is nearly constant from 0·25 to 1·25 full load.

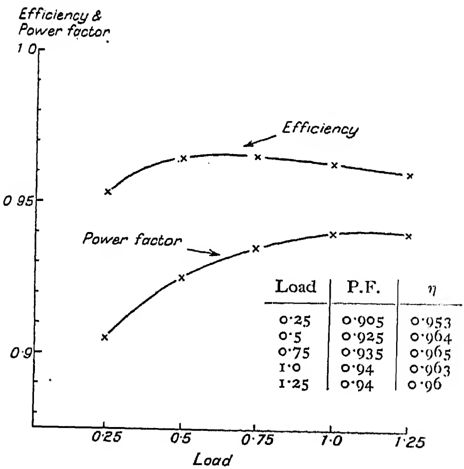
Figs. 1 and 2 show the full-load efficiency with voltage, and the efficiency and power factor with load for a modern mercury arc rectifier of 2,000 kW output at 1,650 volts.

The cathode of a rectifier must be heated to provide an



Volts	η
200	0·87
500	0·925
1,000	0·95
3,000	0·975

FIG. 1



Load	P.F.	η
0·25	0·905	0·953
0·5	0·925	0·964
0·75	0·935	0·965
1·0	0·94	0·963
1·25	0·94	0·96

FIG. 2

electronic emission, but if an attempt is made to increase the current above a certain value, the excessive bombardment by positive ions causes rapid deterioration of a solid cathode, a serious disadvantage not yet overcome. If, however, a mercury pool is used as the cathode, the electronic emission comes from a hot spot on the surface of the mercury and there is no permanent damage done to the cathode by the bombardment of the heavy positive ions.

Mercury is the most suitable substance for the cathode for several other reasons—it is liquid at ordinary temperatures, it has a high atomic weight and a low specific heat, the boiling-point is relatively low, it has a small latent heat of evaporation and the arcing potential is low at low vapour pressures.

When the arc is struck the emitted electrons immediately ionize the mercury vapour molecules and the hot spot is maintained at its electron-emitting temperature by the bombardment of the massive positive ions.

The hot spot on the cathode which gives out a brilliantly white light is not stationary but travels swiftly and erratically over the surface of the cathode pool.

This transitory action is caused by the rapid emission at the cathode hot spot sufficient in the case of small arcs to cause a depression on the mercury surface of as much as 5 mm. with a local static pressure of 10 mm. and a mean temperature of 180° C., and there is a rapid flow of mercury vapour from this spot to the low pressure regions of the container, the velocity of the expanding vapour being of the order of 700 ft. per second. On account of the local rise in pressure the electronic emission diminishes at this spot because the increased local pressure would necessitate an increased general temperature to maintain the emission. Also the increased pressure restricts the free movement of the electrons and they tend not to acquire sufficient velocity to effect ionization, and as the degree of ionization falls off the arc will transfer itself to a near region of lower pressure where ionization is proceeding more rapidly. In this way the arc is continually moving, in an erratic fashion, to find a place of low pressure and high electronic emission.

The surface of the cathode pool is far from quiescent; the sudden generation of localized pressure is accompanied by a depression of the immediate surface of the mercury pool and the rapid irregular transit of this depression, together with the wave reflections from the walls of the pool, give rise to the well-known choppy surface.

From what has been said, if the arc is confined by reducing the area of the cathode pool to a small value, say 0·25 sq. in., then it would appear that the arcing potential must rise. Experiments made by the authors show that this is true, the arc drop rising from 25 volts to 35 volts for a constant current of 9 amperes when the area is so restricted.

It is not sufficient merely to ensure that the cathode provides an electronic emission, it is essential for the proper functioning of the arc that the vapour pressure should lie within moderately well-defined limits.

If the vapour pressure is too low, there will be a reduced probability of collision between the molecules of the vapour and the emitted electrons, so that the electrical conductivity of the vapour will fall due to insufficient ionization; on the other hand if the vapour pressure is too high, the free path of the electrons is restricted, so that they do not acquire a velocity high enough to produce sufficient ionization by collision.

The effective pressures between which it is desirable to operate are from 0·00109 to 0·0885 mm. of mercury, and since the mercury vapour in the container is always saturated the corresponding mean temperatures are 20° C. and 80° C.

The table on page 6 gives the saturated vapour pressure in millimetres of 0° C. mercury for different temperatures both on the Centigrade and Fahrenheit scales.

The temperature of the hot spot has been the subject of considerable speculation and has been variously estimated at from 600° C. to 3,000° C.

It seems probable that this latter figure is much too high although it is possible that the actual arc embraces regions where the temperature is very high and other regions of comparatively low temperature.

The authors constructed a piece of apparatus in which

	Temperature		Vapour pressure mm. of Hg.
	0° C.	32° F.	0·00016
	5	41	0·00026
	10	50	0·00043
	15	59	0·00069
Range for mercury arc rectifier	20	68	0·00109
	25	77	0·00168
	30	86	0·00257
	35	95	0·00387
	40	104	0·00574
	50	122	0·0122
	60	140	0·0246
	80	176	0·0885
	100	212	0·276
	150	302	2·88
	200	392	17·81
	250	482	75·83
	300	572	248·6
	356·7	674	760

(*Physical and Chemical Constants:* Kaye and Laby.)

the hot spot was confined within a circle 0·25 in. in diameter bounded by Pyrex glass uncooled in any way and at one side of the mercury pool was fixed a very thin strip of iron. A current of about 5 amperes was passed, and after a few minutes the tip of the anode, which consisted of a $\frac{3}{16}$-in. diameter rod of silver steel, was actually melted, the necessary temperature being about 1,400° C., whereas at the cathode the Pyrex glass and the thin iron strip were unmarked so that the average temperature of the hot spot must have been below 500° C.

Estimating by the depth of the depression in the mercury surface that the temperature of the surrounding mercury vapour is about 180° C., it would appear that the mean temperature of the actual hot spot is, for a current of 5 amperes, between, say, 250° and 500° C.

A further piece of apparatus was constructed in which the hot spot could be confined to a small area and a fine platinum-iridium couple was arranged to be in the hottest region. With a current of 4 amperes the temperature registered was 370° C., and when the current was raised to 15 amperes the temperature indicated was about 750° C.

It seems reasonable to believe that the average temperature of the hot spot will increase with the current, but it is doubtful if it ever reaches as much as 1,000° C., although on indefinitely small areas it may be higher than this.

The arcing potential varies according to the type of rectifier and the magnitude of the cathode current, being usually about 20 volts, with a possible range of from 15 to 35 volts.

The total drop from anode to cathode is divided into three parts, that at the anode 2 to 8 volts, that at the cathode about 10 volts, and a drop in the arc itself of 8 to 15 volts or more. The drop at the cathode is practically constant for all values of current and vapour pressure, but the anode drop and the drop in the arc itself show considerable variation according to the current, vapour pressure and the mechanical design of the rectifier.

The total anode-cathode voltage drop in a modern rectifier is not absolutely constant but shows a tendency to rise as the current is increased. The cause of this increase is due to increased current density in the arc itself, whose cross-section is limited by the anode and cathode shields. In modern practice the anode shields are made of as large a cross-section as possible, even resorting to oval shapes to attain this object, and the length of the arc is reduced to the smallest possible value in order to cut down the drop in the arc itself.

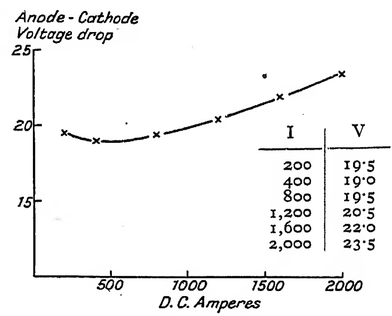
Fig. 3 shows the anode-cathode voltage drop for a Brown-Boveri rectifier, type A 46, rated current 1,000 A., between 200 and 2,000 amperes.

The anode-cathode drop in voltage depends also on the pressure of mercury vapour in the container. When the temperature was 10° C. for a given current the voltage drop was 25 volts, then as the temperature rose the voltage drop gradually fell to a minimum value of approximately 19·5 at 50° C. and then gradually rose again to about 21 at 75° C.

The drop in voltage is greatest when the rectifier is quite cold because there is then far too little ionization. When starting from cold the time required to arrive at the normal

working condition is a matter of 2 to 3 minutes with full load current, and in some cases 'bake out' coils are fitted in the container to shorten the time taken to reach the working temperature.

When a number of anodes are associated with a single cathode as in a multiphase rectifier, the anodes are successively charged positively and negatively and the arc behaves precisely like a stationary brush on a rotating commutator except that the arc (brush) is rotating and the anodes (commutator bars) are stationary.



I	V
200	19·5
400	19·0
800	19·5
1,200	20·5
1,600	22·0
2,000	23·5

FIG. 3

The arc acts like a brush which can convey current in one direction only and which ceases to exist if the current falls below a minimum value of about 3 to 5 amperes, depending on the size and design of the rectifier.

If two adjacent anodes are maintained at exactly the same potential, the flow of electrons from the cathode would divide equally between them provided the anodes were identical in shape and disposition with regard to the cathode and that the external characteristics of the two circuits were identical, but if the potential of one of the anodes rises a very small amount above the other, then all the electrons will immediately try to flow to the anode at the higher potential and none to the other anode. If this instantaneous

transference were to take place, then steady working of the rectifier would be impossible.

Actually, however, as soon as the electron stream begins to transfer from one anode to the other, the inductive drop in the circuit tends to oppose the change, with the result that the electron stream does not transfer instantly, and there is a period of overlap which renders the arc stable.

Quite apart from any consideration of the electric constants of the circuit, ionized mercury vapour persists in the vicinity of an anode for an appreciable interval of time after the voltage has fallen, so that there is an upper limit imposed on the frequency that can be applied to a rectifier which is found in practice to lie between 2,000 and 5,000 cycles per second, frequencies which are outside of most, if not all, of the practical applications of alternating current for industrial purposes.

Back- and cross-firing represent a breakdown of the rectifying action of the arc and are caused by a temporary cathodic action of the anodes. If promiscuous firing occurs at starting when the anodes are cold, it is generally found to be due to small particles of mercury having condensed on the anodes. These small globules of mercury rapidly disappear as the anodes warm up, so that this cause of trouble is soon eliminated.

The anodes are usually of graphite or carbon, since neither of these substances is wetted by mercury and both have negligible electron-emitting properties even when very hot.

In all types of rectifiers it is the practice to shield the anodes from one another and from the cathode. In the glass containers the anodes are situated in side pockets having one or two right-angle bends which are generally effective in preventing both cross- and back-firing even when the anodes become hot.

In rectifiers with metal containers the anodes and cathode pool are fitted with insulating shields which give protection against promiscuous firing, provided they are of ample design and are suitably disposed.

It does not appear that promiscuous firing can be

2

attributed solely to electronic emission from the anodes as shielding alone offers no real obstacle to the passage of electrons, and it would appear that some extra explanation must be sought for this objectional phenomenon.

L. B. W. Jolley, *A.C. Rectification*, gives four possible causes for promiscuous firing, viz. an excessive voltage, an abnormal vapour pressure, impurities in the cathode or quantities of foreign gas. Other writers have attributed this phenomenon to the effects of ultra-violet light from the arc, but in the opinion of the authors this uncontrolled firing may be partly due to the following simple cause: The mean pressure within the container of the rectifier is extremely low and the vapour generated at the small area of the hot cathode spot, at considerable pressure, expands so rapidly that molecules of vapour are projected at high velocity throughout the vacuous space. These molecules of vapour would normally travel in straight lines at a velocity of the order of 700 ft. per second, but owing to the violent turmoil in the container they travel in all directions, and on striking an obstacle they are decelerated and thereby alter the potential of that obstacle with respect to the surrounding parts and consequently set up parasitic currents within the container.

The use of negatively biased control grids of a not too coarse mesh round the anodes is a logical step in the prevention of back- and cross-fires and is more effective than shielding, which can only protect the anodes from the main blast of vapour proceeding in well-defined directions. If, on the other hand, the grid mesh is too fine, then no electrons will reach the anodes and the rectifier will fail to function.

Against the violent eddies and whorls of vapour which exist in any container where comparatively large local temperature and pressure differences exist, shielding alone cannot be regarded as a completely satisfactory safeguard. A charged grid, on the other hand, is proof against the passage of really high-velocity electrons.

The control grids comprise a nickel grating insulated from and in close proximity to each anode and so arranged that no electrons can reach the anodes without first passing

through the meshes of the grid. The grids are provided with vacuum-tight leads to external terminals so that they can be connected up to the negative grid bias control gear.

When an anode becomes positive, but before it actually fires, a negative charge is induced and held on the surrounding grid, and firing of that anode cannot commence until this negative charge is removed by the application of positive grid bias.

With a grid of normal mesh the actual value of the applied negative bias required to prevent firing is quite small, say 2 or 3 volts; in fact, if a grid is merely isolated it will collect a bound negative charge sufficiently great to give it the necessary negative bias to prevent the anode firing under some circumstances. To make the blocking action quite definite under all conditions the grid is given a negative bias not less than 25 volts maintained by means of an external direct-current battery for as long a period as may be desired. No matter what value the positive potential of the anode may rise to, so long as no electrons penetrate the zone between the grid and the anode, the dielectric strength of this space is maintained and no current can flow from the anode.

When at the required instant the grid bias is made positive the negative charge is neutralized and the grid becomes positive so that electrons now penetrate the zone between the grid and the anode and current flows from the anode to the cathode.

The grid thus has the property of inhibiting a flow of current from an anode but can do nothing to modify or interrupt this current once it has started, as is explained later on.

One overriding fact is that the ionization of the mercury vapour tends, naturally, to be confined to a relatively small-diameter tube between the cathode and the anode with the highest positive potential. Complete ionization of the vapour within the container never happens, otherwise an uncontrollable and permanent short circuit would exist.

The current density varies widely in its passage from the anode to the cathode. At the anode the current density may

vary from 10 to 25 amperes per sq. in. of surface, depending upon the type and efficiency of the anode cooling arrangements.

In the arc itself the current density within the anode shields is of the order of 50 to 80 amperes per sq. in., but this current density rises towards the cathode and at the hot spot on the surface of the mercury pool it reaches a very high value, variously estimated at from 11,000 to 25,000 amperes per sq. in.

The reason for this abnormal concentration of current at the cathode is due in some measure to the well-known 'pinch' effect. The heavy, slow-moving positive ions are more concentrated near to the cathode and the magnetic loops surrounding the current exercise more effect on the slow-moving positive ions than on the fast-moving electrons, so that the positive ions are squeezed together. Increasing the current flow increases the magnetic field so that the 'pinch' effect becomes greater and the hot spot does not appreciably increase in size with increased current.

CHAPTER II

CONSTRUCTIONAL DETAILS

AS has been pointed out, steel containers are used for demountable mercury arcs of large output, and a typical example is that shown on page 14 and manufactured by the General Electric Co., England. The main constructional features of the tank are shown in figs. 4 and 5, to which the following description applies:

A Bottom casting.
B Bottom water-jacket.
C Cathode water-jacket.
D Cathode shield.
E Mercury cathode.
F Top casting.
G Vitrified steel insulator
H Anode head.
I Anode seal, head, and stem.
J Anode seal, foot.
K Anode radiator, and stem.
L Anode shield.
M Top casting.
N Top water-jacket.
O Rubber water connexion.
P Vacuum connexion.
Q Striking gear connexion.
R Mercury drip shield.
S Anode mounting centre lines.
T Water inlet.
U Insulated foot.
V Anode stem insulation.
W Water outlet.
X Cable socket.
Y Bottom insulation.
Z Water connexion.

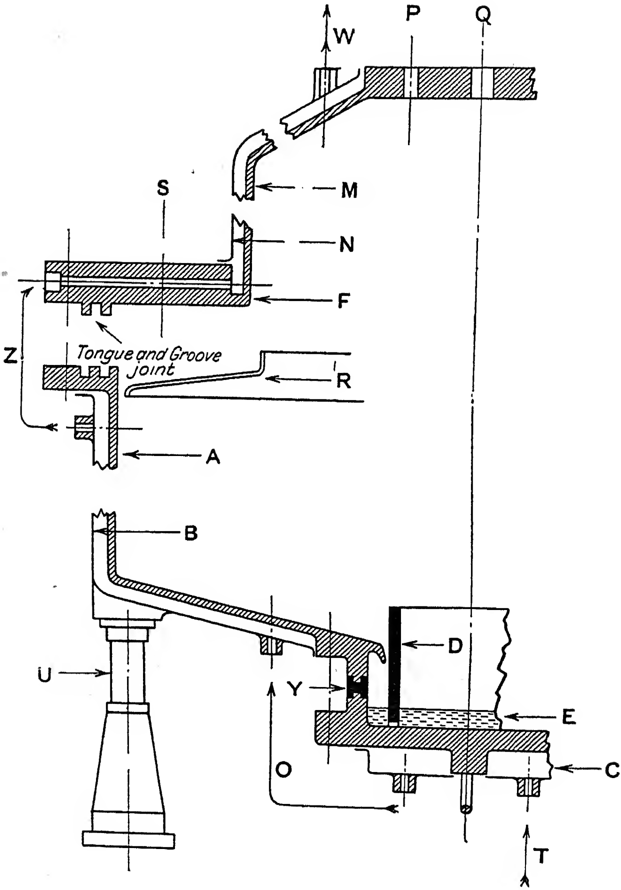


FIG. 4

The cooling water enters at T and keeps the mercury pool cool; it then passes through the rubber connexion O to the bottom water-jacket. Since the mercury pool is insulated from the bottom casting at Y, it is essential that the connexion O should be insulated also.

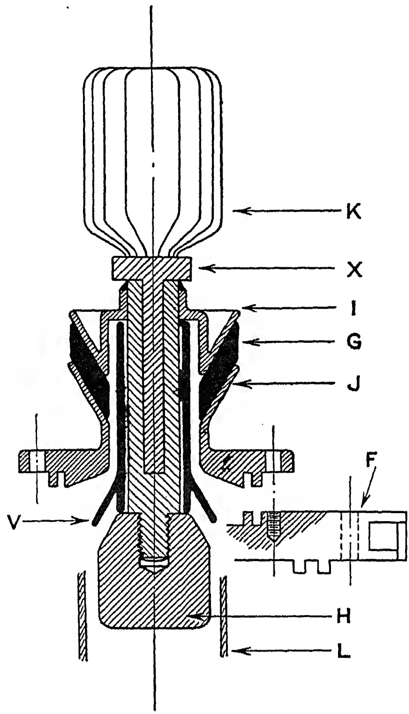


FIG. 5

The water then passes from the bottom water-jacket to the top one via the connexion Z, which is of metal. It should be noted that access to the top water-jacket is got through a series of radial ducts in the top casting F in order that this

part may be adequately cooled. After passing through the top water-jacket the water finally leaves the system at W.

Since the mean temperature of the mercury vapour cannot be allowed to exceed 80° C., the outside surface of the container wall would probably have to be kept down to 50° C., because the temperature gradient through the wall would be about 20° C. and the vapour temperature near the wall would be at least 10° C. lower than its mean temperature.

If simple air cooling were relied on, and the ambient temperature were 15° C., then about 5 sq. in. of cooling surface would have to be provided for each watt loss in the arc.

A rectifier having an output of 1,000 amperes would have an arc loss of approximately 20,000 watts, so that the required surface area of the container would be 100,000 sq. in. and the container would be about 20 ft. diameter and 12 ft. high, which is much too bulky and encloses much more space than is required to accommodate the arc.

Where forced air-cooling is employed the surface area of the container is much smaller, and where the air quantity circulated is 3 cu. ft. per second per kW loss, the surface area is such that the air temperature rises 10° C. in passing over it.

With water cooling, provided that the space between the container wall and water-jacket is not too wide, so that all the water is brought into contact with the hot surface, about 5·75 gallons of water per hour per kW loss would be required to maintain the container wall at 50° C., assuming that the inlet water temperature is 15° C.

For a loss of 20 kW, therefore, about 115 gallons of water per hour would have to be circulated, and this is a relatively small quantity and could be readily passed through a quite small jacket. With water cooling, then, even if the inlet temperature is higher than 15° C., there is no need to make the container any larger than is required by the purely mechanical assembly of the anodes within it, and a rectifier having an output of 1,500 kW at 600 volts would have a container about 6 ft. diameter and 6 ft. high overall.

The function of the drip shield R is to ensure that the mercury vapour condensed at M is distributed over the surface A for further cooling before being returned to the cathode pool E.

Fig. 5 shows the main anode construction, and where auxiliary anodes are used they are of similar construction but of smaller size.

The anode body consists of two steel castings I and J separated by an insulator G. This insulator is formed from a series of thin steel cones vitrified together and to both I and J, so that whilst I and J are thus rendered gas-tight they are effectively insulated from one another. To the top casting I is welded the anode stem which carries the renewable head H, the stem is protected from I and J by means of the sleeve V and carries the air-cooled radiator K and the cable socket X, the radiator stem is screwed or shrunk into the anode stem and there is no possibility of gas leakage at this point.

The complete anode units are mounted on the centre lines S between the radial water ducts in the top casting F and the tongue and groove joint is rendered gas-tight by means of lead gaskets. Mercury seals are sometimes used, but the authors' experience is that unless they are kept clean they are not gas-tight. It should be noted that the tongue and groove joint between the top casting F and the bottom casting A is also rendered gas-tight by means of lead gaskets. Another form of gasket is a steel-covered asbestos ring not unlike those used for motor cylinder sparking plugs. Rubber can be used also, provided it can be kept cool.

The striking gear for starting up the arc from cold consists usually of an inverted steel cup either floating in a mercury bath or spring supported and surrounded by a solenoid coil. The steel cup carries a carbon-tipped steel rod, and when the solenoid is unexcited the carbon tip is just clear of the mercury pool. On exciting the solenoid coil the steel cup is pulled down and the carbon tip dips into the mercury, and when the circuit is broken the tip withdraws and a small arc is formed which causes the auxiliary anode to fire. The auxiliary anodes are usually fed with a unidirec-

tional current so that, once having fired, they will continue to burn and provide an electronic emission sufficient to ensure that the main anodes fire regularly. The auxiliary anodes thus obviate the necessity for continual operation of the striking gear and also enable the main anodes to function on light loads. The solenoid coil, steel cup and mercury bath, or spring supports, are mounted in a case which is attached to the outside of the top casting at Q by means of a gas-tight joint, and if the steel rod is rather long, then it must be provided with a guide supported on the drip shield R.

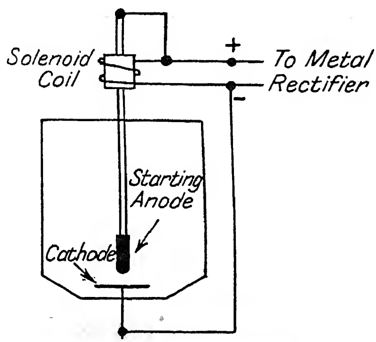


FIG. 6.—Striking Gear

A typical diagram of connexions for the striking gear is shown in fig. 6, and it will be noticed that as soon as the tip of the striking rod touches the mercury pool the current is shunted from the solenoid coil, allowing the rod to rise again and draw out the necessary arc.

It has been pointed out that it is essential to remove from the container practically all traces of any gas other than mercury vapour in order that the mercury may remain clean, that the arc voltage drop may be a minimum and that no wasting of the anodes takes place. Some of the earliest efforts made to produce a high vacuum were the experiments of Otto von Guericke, in the year 1650, who used

a mechanical pump consisting of a cylinder fitted with a plunger and operating on the same principle as the cylinder of a steam-engine which takes steam from the boiler and discharges it into the atmosphere.

It is not possible with a pump of this description to produce a very high vacuum on account of back-leakage past the plunger, but even if the plunger were perfectly tight a complete vacuum could not be obtained for the following reason:

If $a =$ volume of container to be exhausted
$b =$ volume of pump barrel
$d =$ initial density of gas
$d_1 =$ gas density after 1st exhausting stroke

then $d_1 = d\left(\frac{a}{a+b}\right)$

and $d_2 = d_1\left(\frac{a}{a+b}\right) = d\left(\frac{a}{a+b}\right)^2$

so that $d_n = d\left(\frac{a}{a+b}\right)^n$

and d_n can only be zero when n, the number of exhausting strokes, becomes infinite.

With a rotary mechanical pump of correct design such as the 'Hyvac' it is possible, under good conditions, to produce a vacuum as low as 0·0004 mm., and they are used as backing pumps for either mercury or oil condensation pumps when it is required to produce a really low vacuum.

A typical condensation pump in stainless steel is shown in fig. 7 as designed by Burch and Sykes (see Bibliography) and to which the following description applies:

A Backing pump connexion, pressure for oil condensation=0·05 mm.
pressure for mercury condensation=0·2 mm.
B Throat for connexion to arc container.
C Water-jacket for condensation of working fluid, oil or mercury.
D Baffle to restrict back flow into container.

E Cowl to cause down-blast of working fluid vapour.
F Uptake pipe from boiler.
G Baffles to restrict back flow of condensate.
H Boiler for working fluid, pressure for oil=0·1 to 0·2 mm.; for mercury 2 to 3 mm.

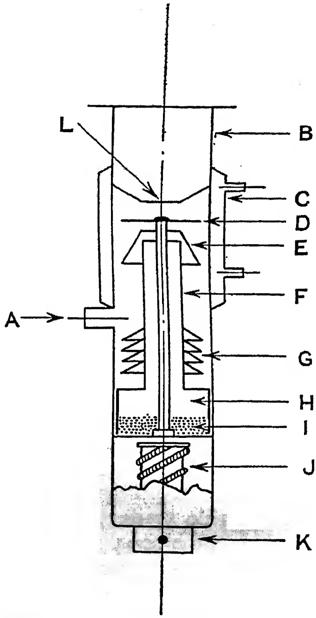


FIG. 7.—Condensation Pump

I Working fluid, oil or mercury.
J Electric heater for boiling fluid.
K Connexion socket for heater.
L Throat restriction to assist prevention of vapour back flow.

The action of the condensation pump is as follows: the electric heater of 250–500-watt capacity boils the working fluid at a temperature between 150° C. and 350° C., depending

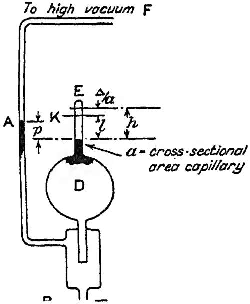
on the fluid. The vapour issuing from the uptake pipe is deflected downwards by the cowl and entrains those gas molecules from the arc container in the vicinity of the down blast. These gas molecules are ejected from the system by means of the backing pump and the working vapour is condensed by the water-jacket and, falling to the bottom of the pump, is returned to the boiler.

In the case of the mercury arc it is usual to use mercury as the working fluid in the condensation pump, and so long as the water-jacket is supplied with a quantity of cold water adequate for the condensation of the mercury vapour no extra cold trap is required between the condensation pump and the arc container. It is advisable to place a trap between the condensation pump and the backing pump to prevent both mercury and water vapour having access to the backing pump and causing a rapid falling off in its performance. Using a mercury condensation pump, it is possible to reduce the total pressure in the arc container to practically that corresponding to the pressure of mercury vapour at the container temperature, which means that all other gases have been removed. It should be noted, however, that where a vessel such as a wireless valve is to be completely exhausted of all gas and vapour the mercury condensation pump must be provided with a cold trap containing liquid air or some such refrigerant, since without such a trap the total pressure in the vessel to be exhausted cannot fall below the vapour pressure of mercury at the water-jacket temperature. Chemical processes are used for the final clean up and are known technically as 'getters'.

On the other hand, an oil condensation pump may not need a cold trap since oils are available which at 350° C. have a vapour pressure not greater than that of mercury at 15° C.

In practice, after working a short while, the vacuum of a mercury arc improves above the value obtainable when cold, due to the fact that the working temperature helps the escape of occluded gas and the scrubbing action of the mercury vapour removes it from the surfaces to which it naturally clings.

The standard piece of apparatus for measuring the very low gas pressures obtaining in the mercury arc container is the McLeod gauge. The principle of the gauge is that a known large volume of the low-pressure gas is compressed into a small known volume with a consequent rise of pressure to a value which can be readily measured. If the temperature of the gas is not changed in the process, then since



Gauge

is raised again, then a volume V of the low-pressure gas will be cut off and driven up into the capillary E under a high pressure which can be measured provided the gauge has been calibrated and a suitable scale is fixed to the capillary tube. The process of calibration (R. J. Clark, see Bibliography) is as follows:

Make an arbitrary mark K below the deformed top of the capillary (see fig. 8). Pump the gauge out, but not completely, then raise the mercury cup and let the difference of level between the mercury columns be p,

let $v=la+\Delta$ where Δ is a constant which allows for the vol. of the capillary above K

$=ha$

so that $h-l=\frac{\Delta}{a}$ and is constant provided the capillary bore is uniform

let V=the total measured volume of the vessel D

P=the permanent gas pressure before compression

then since the saturation pressure of the mercury vapour is the same on both sides of the gauge:

$$PV=(P+p)ha$$

and

$$P=\frac{ph}{\frac{V}{a}-h} \qquad \text{(1)}$$

Now h is measured from a point above K by $\frac{\Delta}{a}$ and h can be measured directly if K is arranged to be $\frac{\Delta}{a}$ below the division O on the capillary scale.

To find the magnitude of $\frac{\Delta}{a}$, first set the scale division O opposite the mark K and reduce the pressure in the gauge by pumping it out to about 0·01 mm. Now raise the mercury cup C and compress the gas into the capillary and evacuate the gauge as completely as possible.

Take one reading with the meniscus in the open capillary

p above that in the closed one and let the closed capillary meniscus be l below K. Readjust the position of the cup C and take a second reading, letting p_1 and l_1 be the observed values.

If the evacuation of the gauge is complete, then p is the only pressure on the gas in the closed capillary tube and its volume is $la+\Delta$, and since the original conditions of the permanent gas were P and V therefore

$$PV=p(la+\Delta)=p_1(l_1a+\Delta)$$

thus $$p(la+\Delta)=p_1(l_1a+\Delta)$$

so that $$\frac{\Delta}{a}=\frac{pl-p_1l_1}{p_1-p}$$

then set the zero scale reading O above the mark K by the amount $\frac{\Delta}{a}$. Note that, although the mark K can be set anywhere below the closed end of the capillary, the origin of h is not arbitrary. The gauge is now ready for use and if, when taking a reading $p=h$ by always raising the mercury cup C until the open capillary meniscus is level with the scale reading O, then from (1)

$$P=\frac{h^2}{\frac{V}{a}-h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and is the pressure due solely to permanent gases such as air, &c., since the saturation pressure of the mercury vapour is equal on both sides of the gauge under all conditions.

The following example due to Mr. Clark will demonstrate the relative sizes of the various parts of the gauge as required by the pressures to be measured:

Measured total volume of vessel D $=V=50{\cdot}421$ c.cm. at 16° C.
Cross-sectional area of Capillary E $=a=0{\cdot}017647$ sq. cm.

So that $\frac{V}{a}=2857{\cdot}2$ cm.

Measured value of $\frac{\Delta}{a}$		$=0{\cdot}0387$ cm.
Measured length of capillary E		$=25$ cm.
Maximum pressure readable	P	$=2{\cdot}207$ mm.
For 1 cm. scale reading	P	$=0{\cdot}00350$ mm.

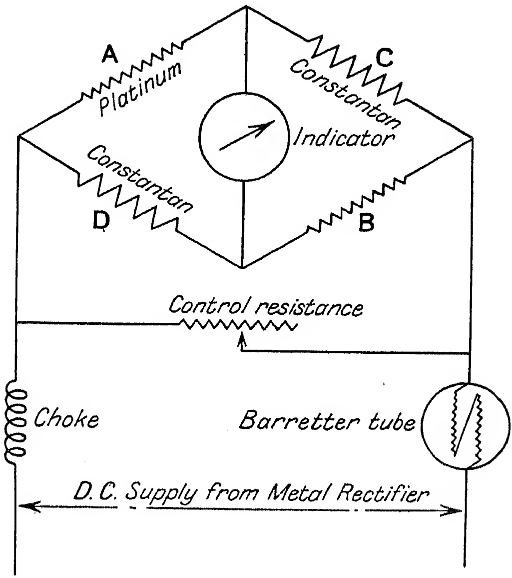


FIG. 9

The McLeod gauge is not very suitable for permanent operation since the mercury will inevitably become dirty, the walls of the capillaries fouled and the meniscus indefinite and sluggish. Its principal use is, when freshly made up, to act as a standard for the calibration of more robust devices like that known as the Pirani gauge and also described in *E.T.Z.* 1906 as Kallmann's Narrow Range Voltmeter.

The Pirani gauge, as shown in fig. 9, is made up in the

form of a Wheatstone Bridge having two arms of platinum resistance wire and two of constantan. The bridge is fed from a metal rectifier and the galvanometer is graduated tc read the gas pressure directly, having been calibrated by means of a McLeod gauge. The D.C. supply from the smal metal rectifier is smoothed by means of a choke coil and the current is maintained constant by means of a Barretter tube probably so named after Professor Barrett.

A Barretter tube consists of a thin spiral of iron wire enclosed within a glass envelope which contains an atmo-sphere of hydrogen gas. The dimensions of the wire are such that it glows at a dull red heat and the hydrogen is to prevent oxidation of the wire.

Under these conditions and within certain limits of P.D variation the resistance of the wire increases almost in direc proportion to the P.D. so that the current, within thes limits, is constant.

The two platinum arms of the bridge are enclosed in gas-tight box connected to the mercury arc container b means of tubing, and it is necessary, if the bridge is to wor satisfactorily, to ensure that mercury vapour does not com in contact with the platinum wires, and a cold trap betwee the box and the container could be used for this purpose The cold trap will not, however, prevent the low-pressur permanent gases having access to the box.

The two constantan arms are exposed to the atmospher and constantan has the property of almost constant resist ance through a wide temperature range. Platinum, on th other hand, has a temperature coefficient of 0·0034 c.g.s. an is very resistant to chemical action, but may absorb gas whe heated. Metals other than platinum can be and are used i the construction of the Pirani gauge.

Thermodynamic theory shows that the heat conductivit of a permanent gas is a function of its pressure. In a absolute vacuum the conductivity is zero and it rises in stri proportion to the gas pressure through a certain range of lo pressure which covers the requirements of mercury a working. With increasing pressure, however, the condu tivity tends to a constant limiting value which is reached

a gas pressure of about 1·0 mm. of mercury. If, then, the platinum wires are arranged to carry a constant current in the presence of permanent gas of low pressure, they will attain a temperature and a resistance proportional to the degree to which the pressure has been reduced.

If the P.D. across the bridge is maintained constant, then, as is shown in the Appendix, the galvanometer deflection is proportional to the ohmic resistance of the platinum arms, that is, to the vacuum.

It is, of course, necessary to calibrate the scale of the indicator by means of a McLeod gauge at several points, then, owing to the strict proportionality between gas pressure and the resistance of the platinum arms of the bridge, it is probable that the Pirani gauge could be made to give reliable readings for lower pressures than could be conveniently measured on a McLeod gauge. One arm only of the bridge could be made of platinum and exposed to the gas pressure if desired, but the double arrangement shown has the advantage of giving a large reading on the indicator. The function of the control resistance shown in fig. 9 is to assist in maintaining the Barretter current at the correct working value, which is about 0·3 amperes. The indicator scale is graduated in microns and one micron is a pressure of 0·001 mm. of mercury.

CHAPTER III

GRID CONTROL OF VOLTAGE

AN outstanding feature of modern electrical engineering development is the mercury arc rectifier, and its advent is due, almost entirely, to the fact that its operation is susceptible to grid control.

Suggestions for the control of electronic emission, by means of grid bias, were first made by Dr. L. De Forest in about the year 1913, but it was not until about 1927 that successful vacuum seals were produced which could be applied to arcs of moderately large output and a grid controlled arc of 500 kW output was built in Switzerland.

The grid consists of an envelope of perforated sheet steel quite close to and enclosing the anode in such a way that the arc, when formed, must pass through the relatively coarse meshes of the grid. The grid is, of course, insulated from the anode and is provided with a lead which is brought out through a suitable seal and each anode is provided with a separate grid.

As in the thermionic valve, the striking potential of the mercury arc can be controlled by means of biased grids placed between the anodes and the cathode.

In the valve, however, the enclosing vessel is almost entirely free from ionized vapour and in consequence a grid retains its P.D. with respect to the cathode and its power of controlling the emission under all conditions of current flow. In the mercury arc the enclosing vessel contains ionized mercury vapour which surrounds a negative grid immediately the arc is struck, tends to suppress the grid-cathode P.D., and thereby robs the grid of any further control until the arc dies out.

Thus, in the mercury arc, the grid bias controls the striking potential and hence the instant at which striking takes place, but it has no further control over the emission,

or current flow, once the arc is struck. The grid is maintained at a potential negative to the cathode by means of a battery, or other source of direct current, and although this potential is not critical in value it is, to ensure effective blocking, generally between 25 and 250 volts depending on the voltage rating of the rectifier.

To ensure instantaneous starting of the arc at the desired point in the cycle the grid bias is changed from negative to positive, the positive value being probably as much above the datum line as the negative value was below it.

In further explanation of grid control it should be remembered that, before an arc is formed, a positive anode would induce and bind a negative charge on an isolated grid and that electrons would not, in general, pass to the anode because the attractive force of the anode would be balanced by the repulsive action of the grid.

If, however, the grid mesh is open, then electrons may be shot through by their explosive emission from the cathode unless the grid is made definitely negative, with respect to the cathode, by means of a battery. The minimum value of the necessary negative bias is small, a few volts being all that is required, but to make the blocking action of the grid absolutely definite it is usual to employ a minimum negative bias of about 25 volts.

It is essential, for the arc formation, that all the negative bias be removed and a momentary positive bias applied at the instant at which it is desired to strike the arc, and in general a positive bias equal to the original negative bias is employed.

It is usual to cause the bias to change over quite suddenly, giving what is known as impulse, or hard, control, and there are two well-known methods of securing this change over, namely, battery control and saturated, or peaky, transformer control.

In fig. 10, which represents one phase of a multiphase arc, the rotating member of the 'make and break' is driven synchronously and the instant, at which the grid bias is made positive with respect to the cathode, is varied by the position of the fixed member with respect to the rotating member.

When the contact is open the grid is negative to the cathode by A volts and when the contact is closed the current is $i=\dfrac{A+B}{R_1+R_2}$ amperes and the grid is positive to the cathode by $\left[\left(\dfrac{A+B}{R_1+R_2}\right)R_2-A\right]$ volts.

Fig. 11 shows one phase of a tri-phase arc controlled by saturated, or peaky, transformer grid bias and the total number of transformers required is equal to the number of anodes fitted.

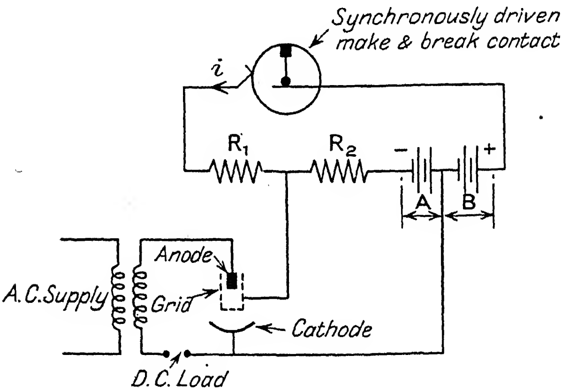


Fig. 10.—Battery Control

By using two transformer windings, one star and one mesh connected, two fluxes 150 degrees apart are obtained and their magnitudes can be controlled by the resistances. The time phase of the positive A.C. component of the grid bias is thus controlled as shown by the angle α, fig. 12.

This positive A.C. component is made peaky by the simple expedient of working the grid transformer core at a high degree of saturation.

If the grid were not negatively biased, commutation from one anode to the next would tend to take place at the point A.

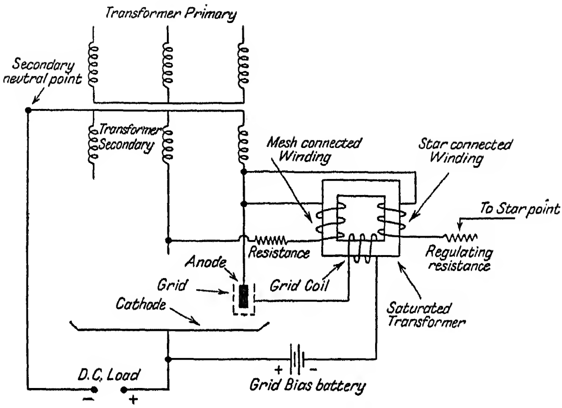


FIG. 11.—Transformer Control

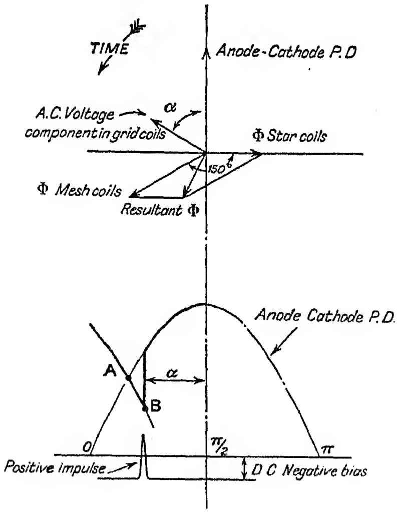


FIG. 12

If the grid had no positive A.C. component to cause positive bias, commutation would probably never take place, but with this component it takes place at B, fig. 12.

If soft control is required, then a sine wave A.C. component of the grid bias is employed instead of the peaky component.

The manner in which the mean value of the D.C. voltage may be controlled by means of retarding the point at which commutation commences is, for the case where the voltage waves are undistorted as on no-load, most easily demonstrated as follows :

Note.—The following formulae do not hold for the special case of $m=1$.

Fig. 13 shows the no-load E.M.F. waves of an m phase (m anodes) rectifier where, with no grid bias, commutation commences at A ends at B. The mean value of the voltage on the D.C. side is given by

$$\text{Mean D.C. voltage} = \frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}+\frac{\pi}{m}} \mathrm{E}\sin\theta . d\theta$$

$$= \mathrm{E}\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right) . \qquad (3)$$

If commutation is retarded by an angle σ, using negative bias with positive impulses, then the commutating points are at C and D, and if the magnitude of the angle σ is unrestricted, as is the case when rectification is incomplete, then

$$\text{Mean D.C. Voltage} = \frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}+\sigma}^{\frac{\pi}{2}+\frac{\pi}{m}+\sigma} \mathrm{E}\sin\theta . d\theta .$$

$$= \mathrm{E}\left[\frac{m}{\pi}\sin\frac{\pi}{m}\cos\sigma\right] \qquad (4)$$

The mean D.C. voltage is, therefore, a cosine function of σ and is shown by the full line curve in fig. 14.

If rectification is complete, then the angle σ is restricted in the sense that the upper integration limit can never exceed π since the anode-cathode P.D. then goes negative and the arc is extinguished.

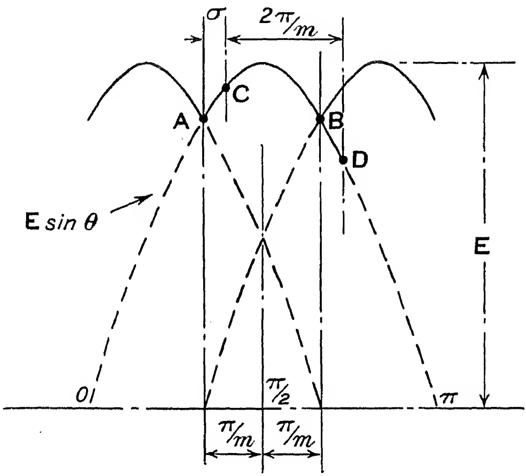


FIG. 13

Where rectification is complete, the mean D.C. voltage curve will follow the incomplete case until σ reaches a value such that $\frac{\pi}{2}+\frac{\pi}{m}+\sigma=180°$ and the point D, fig. 13, reaches π, after which it will conform to the following law and become zero when $\frac{\pi}{2}-\frac{\pi}{m}+\sigma=180°$ and the point c reaches π.

$$\text{Mean D.C. Voltage}=\frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}+\sigma}^{\pi} \text{E} \sin \theta . d\theta .$$

$$=\text{E}\left[\frac{m}{2\pi}\left\{1-\sin\left(\sigma-\frac{\pi}{m}\right)\right\}\right] \quad . \qquad (5)$$

and is shown, partly by the full line and partly by the dotted line, in fig. 14, which figure shows the manner in which the value of the mean D.C. voltage changes as commutation is delayed for $m=6$.

Note.—The complete and incomplete rectification curves are alike until $\sigma=60°$ and the complete rectification curve reaches zero when $\sigma=120°$.

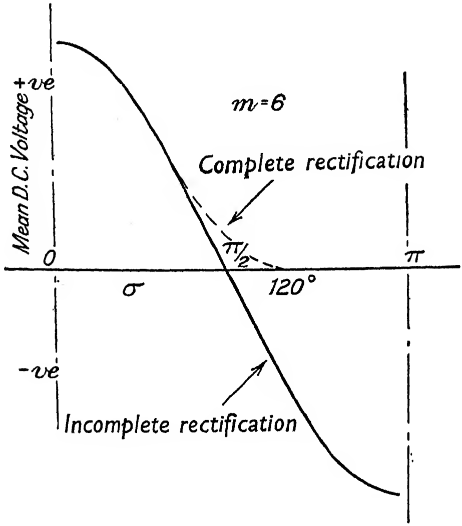


FIG. 14

Equations (4) and (5) are both numerically equal to

$$E\left[\frac{m}{\pi}\sin^2\frac{\pi}{m}\right] \text{ for, } \frac{\pi}{2}+\frac{\pi}{m}+\sigma=180°$$

and equation (5) is zero when $\dfrac{\pi}{2}-\dfrac{\pi}{m}+\sigma=180$

Voltage control by means of retarded commutation is objectionable on account of the extra distortion it produces in the voltage wave (F. J. Teago, *see* Bibliography).

Fig. 13 shows the shape of the no-load E.M.F. wave for an 'm' phase rectifier where m is any number other than unity.

The E.M.F. is that due to the full line tips of the sine curves and, for the case where commutation is unretarded, the mean voltage on the D.C. side has been shown equal to

$$E\left[\frac{m}{\pi}\sin\frac{\pi}{m}\right]$$

so that the no-load ratio mean/maximum voltage is

$$\frac{m}{\pi}\sin\frac{\pi}{m}$$

The no-load ratio R.M.S./maximum voltage is

$$\sqrt{\left[\frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}+\frac{\pi}{m}}\sin^2\theta.d\theta\right]}$$

$$=\sqrt{\left[\frac{1}{2}+\frac{m}{4\pi}\sin\frac{2\pi}{m}\right]}$$

For the special case where $m=1$, that is, a single phase or single anode arc, the current flows in the load circuit

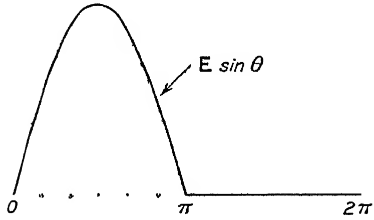


FIG. 15

only during the positive half of each cycle, so that a voltmeter placed across the load would have an E.M.F. wave as shown in fig. 15 impressed upon its terminals.

In this case, then, the ratio mean/maximum voltage is

$$\frac{1}{2\pi}\int_0^{\pi} \sin\theta.d\theta = \frac{1}{\pi}$$

and the ratio R.M.S./maximum voltage is

$$\sqrt{\left[\frac{1}{2\pi}\int_0^{\pi} \sin^2\theta.d\theta\right]} = \frac{1}{2}$$

The numerical values of these ratios, for various values of m, are as set out in the following table:

m	Mean/ Maximum	R.M.S./ Maximum
1	0·318	0·5
2	0·636	0·707
3	0·825	0·840
6	0·955	0·956
m	$\frac{m}{\pi}\sin\frac{\pi}{m}$	$\sqrt{\left[\frac{1}{2}+\frac{m}{4\pi}\sin\frac{2\pi}{m}\right]}$

CHAPTER IV

GRID CONTROL OF CURRENT

IT follows, from the fact that the mean D.C. voltage can be controlled by grid bias, that the magnitude of the current in the D.C. circuit can also be controlled. Probably the most important application of grid control is the limitation of the current which can flow on short circuit by an automatic increase in the angle θ, fig. 16. To render a study of this point as simple as possible consider a short-circuited arc having a single anode, assume that the voltage drop across the arc is negligibly small, and that the angle θ is controlled by means of positive grid bias, as shown in fig. 16.

The positive impulse to the grid bias may be regarded as a switch which closes the circuit at a point whose angular displacement from the origin is θ, the magnitude of θ depending upon the instant at which the positive bias is applied.

The differential equation to such a circuit is, neglecting the primary side of the transformer,

$$E \sin \theta = ri' + x\frac{di'}{d\theta} \quad \text{where } x = 2\pi f L$$

and the solution to this equation consists of the following two terms:

the permanent current

$$\frac{E}{\sqrt{(r^2+x^2)}} \sin(\theta-\beta) \quad \text{where } \beta = \tan^{-1}\frac{x}{r}$$

and the transient current $\quad A\varepsilon^{-a\theta} \quad$ where $a=\dfrac{r}{x}$

so that the total current is

$$i' = I \sin(\theta-\beta) + A\varepsilon^{-a\theta} \quad . \quad . \quad . \quad . \quad (6)$$

Where A is a constant, the value of which depends upon the terminal conditions of the circuit, and I is the maximum value of the permanent current in the circuit.

In this particular case the total current must be zero at the instant of closing the switch so that at this instant

$$A\varepsilon^{-a\theta} = -I \sin(\theta - \beta) \quad . \quad . \quad . \quad . \quad (7)$$

thus the minimum transient occurs when $\theta = \beta$

and the maximum transient when $\theta - \beta = \frac{\pi}{2}$

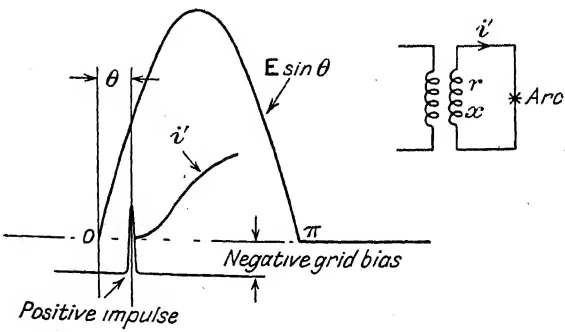


FIG. 16

To make the case definite let E=100 volts, $r=5$ ohms, $x=15$ ohms, and first consider the case where the terminal condition is

$$i' = 0 \quad \text{when } \theta = 0$$

then from equation (7)

$$A = I \sin \beta$$

Now $\quad \beta = \tan^{-1} \frac{15}{5} \quad \therefore \beta = 72^\circ$ and $\sin \beta = 0{\cdot}95$

Also $\quad I = \frac{100}{\sqrt{(5^2+15^2)}} = 6{\cdot}33$ amperes

so that A=6·0.

By evaluating equation (6) the following table is obtained for the case where $i'=0$ when $\theta=0$:

θ radians	Amperes	
	Permanent	Transient
0	$-6{\cdot}0$	$6{\cdot}0$
3	$+6{\cdot}23$	$2{\cdot}21$
6	$-6{\cdot}3$	$0{\cdot}81$

This table has been plotted as fig. 17 and from this it is seen that, when the positive grid bias is arranged so that the switch closes at $\theta=0$, the total current i' rises to a maximum value of 8·8 amperes.

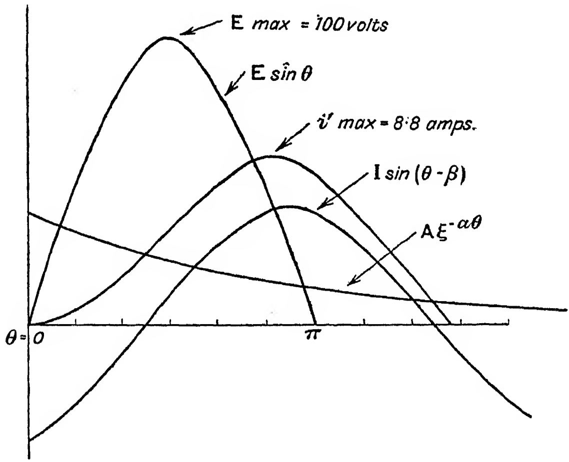


FIG. 17

Now consider the positive bias arranged so that the circuit is closed at the instant $\theta=\beta=72°$. So that $i'=0$ when $\theta=72°$.

Under these conditions equation (7) shows that A is zero and that there is no transient current, which is to say that

for $\theta=72°$ the total current becomes identical with the permanent current, so that

$$i'=I \sin(\theta-\beta)$$

and is shown in fig. 18 to have a maximum value of 6·33 amperes.

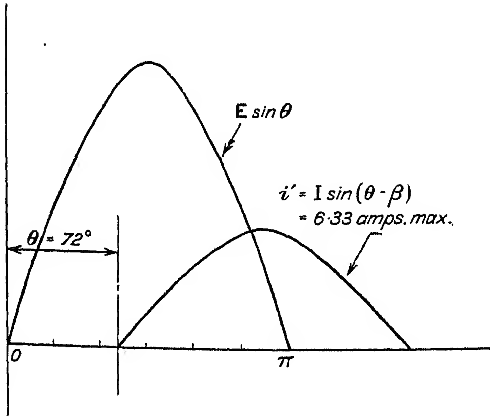


FIG. 18

If the positive impulse is further adjusted so that

$$\theta-\beta=\frac{\pi}{2}$$

i.e. $\theta=162°=2{\cdot}83$ radian

and $i'=0$, when $\theta=162°$

then equation (7) becomes

$$A\varepsilon^{-\frac{2\cdot83}{3}}=-6{\cdot}33$$

So that

$$A=-16{\cdot}3$$

By evaluating (6) the following table is obtained for the case where $i'=0$ when $\theta=162°$ or $\theta-\beta=\frac{\pi}{2}$.

θ radians	Amperes	
	Permanent	Transient
2·83	6·33	−6·33
4·5	−0·66	−3·63
6·0	−6·3	−2·2

This has been plotted as fig. 19, from which it is seen that the total current i' now has a maximum value of 0·3 amperes.

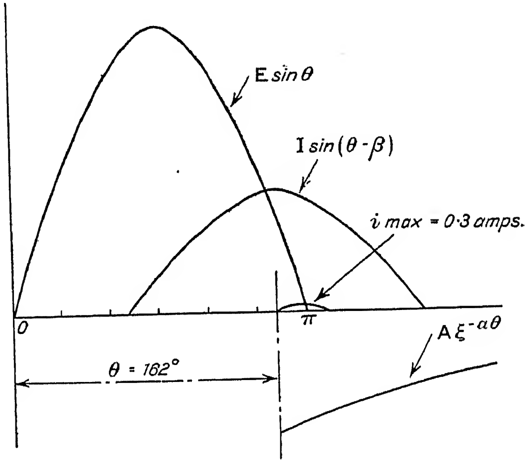


FIG. 19

These figs. 17, 18 and 19 show very definitely how an increase in the angle θ, fig. 16, progressively limits the magnitude of the current which can flow and hence its importance in the suppression of a short-circuit current.

A case of special interest is that in which the resistance of the circuit is zero. For r=zero equation (6) becomes

$$i'=\text{I} \sin\left(\theta-\frac{\pi}{2}\right)+\text{A}$$

$$=\text{A}-\text{I}\cos\theta$$

CHAPTER V

VOLTAGE REGULATION

WHEN a mercury arc is on load the terminal voltage must fall below the no-load value owing to the resistance and leakage reactance drops on the transformer primary and secondary windings, the drop on the anode choke and cathode smoothing coil as well as that on the arc itself. It is very desirable that the natural voltage regulation of the transformer should be close in order that the fall of voltage with load may be kept small. It is possible, of course, to compensate this voltage drop in several ways and the most obvious one is to use an induction regulator in the primary winding of the transformer, but this adds to the cost of the equipment. It is also possible to keep a certain amount of voltage in hand by having retarded commutation and reducing this retardation as the load comes on, but this has the disadvantage of a relatively poor D.C. wave shape on light loads. Tap changing on the primary side of the transformer is also a possible solution.

If, then, it is possible to keep the natural voltage regulation of the transformer close this should be done.

In fig. 21 is shown a transformer with a star-wound primary and secondary giving a tri-phase arc.

When one arc is burning alone a secondary phase such as *ab* is loaded and the other two are unloaded. Mutual inductance between *ab* and AB causes the impedance of the primary phase AB to fall whilst that of the phases AC and AD remains high due to lack of current in the unloaded secondary phase. Now some current flows in all three phases on the primary side at all times and, in consequence, the voltages on AD and AC are much higher than that on AB. This is usually called 'slipping of the neutral' because the neutral moves along AB and may, in an extreme case, reach the point B. It is obviously a source of great weakness to

have to load between the line and neutral point as is done, of necessity, on the secondary side of a multi-phase arc and it is productive, firstly, of very poor voltage regulation and, secondly, of more or less distortion of the primary supply current.

The tendency of the neutral point to slip can be largely overcome by fitting an extra mesh-connected tertiary winding to the transformer, but this again adds to the cost. The tertiary winding acts by reason of the fact that, being mesh connected, third harmonic current induced in any one

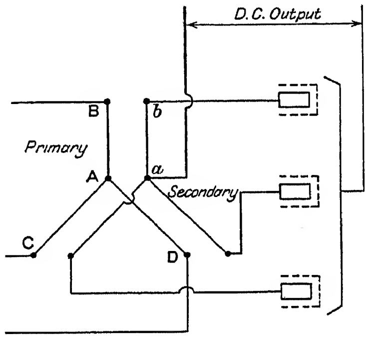


FIG. 21

tertiary leg must flow in all three tertiary legs and, therefore, must cause mutual induction with all three primary phases and must tend to keep their impedances equal and stabilize the neutral point.

It is well known that the voltage regulation of a three-phase transformer is much closer when loaded between two lines than when loaded between line and neutral and fig. 22 shows why this is so.

If a transformer is loaded between line and neutral it is possible for the neutral point to slip from o to o′, but if loaded between two lines it is only possible to slip from o to o″ and this amount is only 50 per cent. of the former amount. In fact, the slipping is so severe with the connexions

shown in fig. 21 that it may be an impossible scheme without a tertiary winding.

Now with a six-phase, or hexaphase, secondary winding on the transformer it is possible, by adopting the triple-star

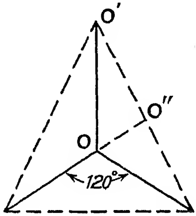


FIG. 22

or fork connexion shown in fig. 23, to load between two lines and thus secure the advantage of the superior voltage regulation inherent in this connexion.

In fig. 23 the anodes are connected to a_1, a_3, b_1, b_3, c_1, and c_3, and a_5, b_5, and c_5 form the common neutral point, the

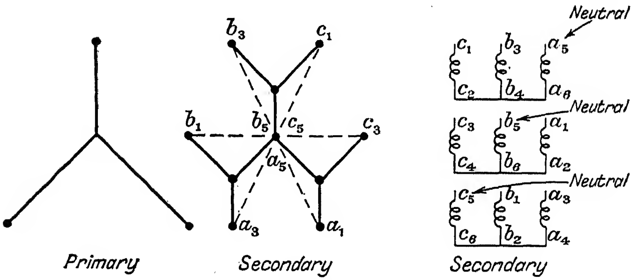


FIG. 23

whole system giving the ordinary hexaphase arrangement with the special distinction of permitting the load to be taken between two lines such as a_1, a_5, &c.

For three-phase working the équivalent secondary connexion is the interconnected star winding.

The voltage regulation between no load and full load

may be approximately estimated by estimating the mean voltage drops in the various parts of the circuit. If the total mean drop is added to the required mean value of the D.C. output voltage, then by using the appropriate value of the ratio mean/maximum voltage it is possible to estimate the required maximum value of the secondary A.C. potential difference between any one line and the neutral point of the transformer.

The mean voltage drops cannot be estimated very closely for several reasons, the chief ones being that:

(*a*) during the commutation period two anodes are firing, whilst in the intervals between commutation only one anode is firing;

(*b*) the current waves follow no simple mathematical law and their shape varies with the number of anodes fitted to the rectifier.

In the following method, where commutation is unretarded, a sine law has been assumed for the current i_1 from any one anode and the mean rectified current i is taken to have a constant value equal to the maximum value of the current from one anode. This constancy of the mean rectified current necessitates that from one anode the current should, in reality, commence to flow at the point A and should not cease at B but should, due to inductance, overlap by some angle θ_0 as shown in fig. 24.

It should also be noticed that the frequency of the current i_1 is, compared with the fundamental, $\frac{m}{2}$, so that

$$i_1 = i \sin \frac{m\theta}{2}$$

From fig. 24, then, it is seen that the mean value of the voltage drop on the resistance r_1 is

$$\frac{m}{2\pi}\int_0^{\frac{2\pi}{m}} r_1 i \sin \frac{m\theta}{2} . d\theta$$

$$= \frac{2}{\pi} r_1 i$$

The instantaneous E.M.F. due to x_1 is (see fig. 24)

$$x_1\frac{di_1}{d\theta}=\frac{mx_1i}{2}\cos\frac{m\theta}{2}$$

and, since only the positive parts of this E.M.F. wave are effective on the D.C. side, the mean value of this is

$$\frac{m}{2\pi}\int_0^{\frac{\pi}{m}}\frac{mx_1i}{2}\cos\frac{m\theta}{2}.d\theta$$

$$=\frac{mx_1i}{2\pi}$$

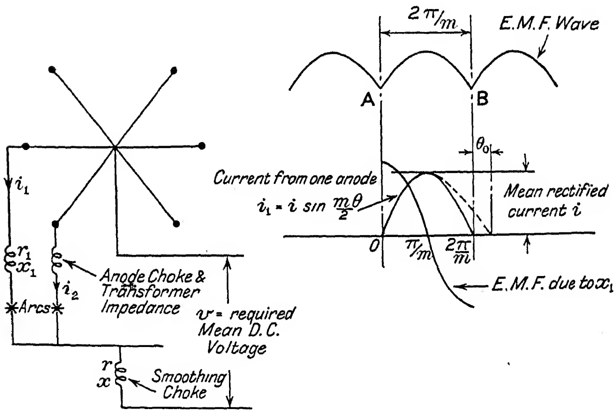


FIG. 24

The mean output current i being constant, for the smoothing choke the

Mean drop$=ri$

Thus the sum of the total mean drop and the mean value of the output voltage is

$$\text{Arc drop}+\frac{2}{\pi}r_1i+\frac{mx_1i}{2\pi}+ri+v$$

and the maximum value of the secondary A.C. potential

difference between one line and the neutral point of the transformer is, for unretarded commutation,

$$E = \frac{\text{Arc drop} + \left[\frac{2}{\pi}r_1 i + \frac{m}{2\pi}x_1 i + ri + v\right]}{\frac{m}{\pi}\sin\frac{\pi}{m}} \quad . \quad . \quad (8)$$

CHAPTER VI

CURRENT—VOLTAGE CHARACTERISTICS

FOR any one anode of an '*m*' anode mercury arc, *m* being a large number,

let E $\cos\theta$ = the A.C. potential difference between the anode and cathode

r = the resistance in the circuit

x = the reactance in the circuit

v = the average value of the required D.C. voltage

and let v vary from $v=E$ when the arc is on open circuit to $v=$ zero when the arc is on short circuit and current may flow over the whole period $0-2\pi$

then the differential equation for the circuit is

$$E\cos\theta = ri' + x\frac{di'}{d\theta} + v$$

with the solution

$$i' = A\varepsilon^{-a\theta} - \frac{v}{r} + \frac{E}{z}\cos(\theta - \alpha)$$

where i' = the instantaneous value of the current in the circuit

$$a = \frac{r}{x},\ z = \sqrt{(r^2 + x^2)},\ \alpha = \tan^{-1}\frac{x}{r}$$

and A is a constant the value of which depends upon the terminal conditions.

The current is seen to consist of three parts as follows

(*a*) the transient term
(*b*) the counter term
(*c*) the permanent term

As shown in fig. 25, if the current flows from any one anode over the period 2λ, starting at $\theta = -\sigma$, then since the current cannot commence to flow until the alternating P.D.

reaches a value equal to the average value of the D.C. voltage therefore

$$v = E\cos\sigma$$

or

$$\frac{v}{E} = \cos\sigma \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Also since conduction begins when $\theta = -\sigma$, then $i' = 0$ when $\theta = -\sigma$

thus

$$A\varepsilon^{a\sigma} - \frac{v}{r} + \frac{E}{z}\cos(\sigma+\alpha) = 0$$

so that

$$A = \varepsilon^{-a\sigma}\frac{v}{r} - \varepsilon^{-a\sigma}\frac{E}{z}\cos(\sigma+\alpha)$$

and

$$i' = -\varepsilon^{-a(\theta+\sigma)}\frac{E}{z}\cos(\sigma+\alpha) - \frac{E}{r}\cos\sigma(1-\varepsilon^{-a(\theta+\sigma)}) + \frac{E}{z}\cos(\theta-\alpha) \quad . \quad . \quad . \quad . \qquad (10)$$

From this equation, for any specified value of σ and by giving θ values equal to, or greater than $-\sigma$, the i' curve in fig. 25 can be plotted.

From equation 10, if the voltage regulation of the supply transformer is neglected so that E is constant, it is easy to determine the connexion between the average value of the

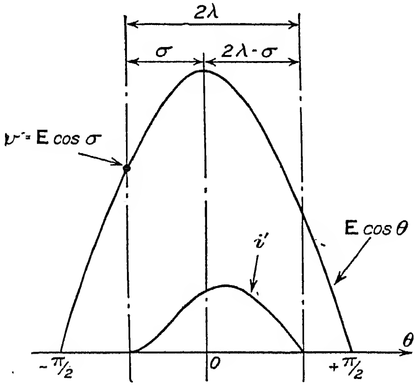


FIG. 25

D.C. voltage and the current flowing between the limits no-load and short circuit for the two limiting cases of no secondary resistance and no secondary reactance.

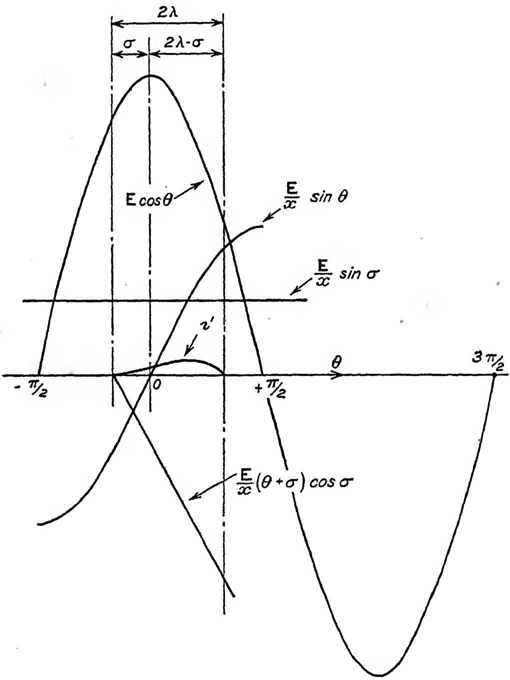


FIG. 26

For r=zero equation 10 becomes

$$i' = \frac{E}{x}[\sin\sigma - (\theta+\sigma)\cos\sigma + \sin\theta] \quad . \quad . \quad (11)$$

noting that the second term in equation 10 takes the form $\frac{0}{0}$ and therefore both the numerator and denominator must be differentiated with respect to r in the usual way.

Fig. 26 shows equation 11 in graphical form, the current i' together with its three component parts.

On no-load both σ and $2\lambda-\sigma$ are zero since $v=E$ whilst on short circuit since $v=0$, $\sigma=+\frac{\pi}{2}$ and $2\lambda-\sigma=\frac{3\pi}{2}$, so that $2\lambda=2\pi$ and it is necessary to determine the relationship between σ and λ from no-load to short circuit.

This is done from equation 11, using the terminal condition

$$i'=0 \text{ when } \theta=2\lambda-\sigma, \text{ fig. 25}$$

so that

$$\sin\sigma+\sin(2\lambda-\sigma)=2\lambda\cos\sigma$$

from which

$$\tan\sigma=\frac{\lambda-\sin\lambda\cos\lambda}{\sin^2\lambda} \quad . \quad . \quad . \quad . \quad (12)$$

and the connexion between σ and λ for r=zero is shown graphically by curve A, fig. 27, which is probably of the correct shape since, with zero resistance, the current flows from 0 to 2π and λ must increase faster than σ, see fig. 26.

The mean current output from the anode, over the period $0-2\pi$, is from equation 11

$$\frac{E}{2\pi x}\int_{-\sigma}^{2\lambda-\sigma}[\sin\sigma-(\theta+\sigma)\cos\sigma+\sin\theta]d\theta$$

$$=\frac{E}{2\pi x}\left[\theta\sin\sigma-\left(\frac{\theta^2}{2}+\theta\sigma\right)\cos\sigma-\cos\theta\right]_{-\sigma}^{2\lambda-\sigma}$$

but the mean current output from one anode, over the period, is also $\frac{i}{m}$, where i is the total current on the D.C. side from all m anodes so that the total mean current output is, from all the anodes,

$$i=\frac{mE}{\pi x}[\lambda\sin\sigma-\lambda^2\cos\sigma+\sin\lambda\sin(\lambda-\sigma)]$$

and $i_{sc}=\frac{mE}{x}$ for a complete short circuit where $\sigma=\frac{\pi}{2}$ and

$$\lambda=\pi$$

so that

$$\frac{i}{i_{sc}}=\frac{1}{\pi}[\lambda\sin\sigma-\lambda^2\cos\sigma+\sin\lambda\sin(\lambda-\sigma)] \quad . \quad (13)$$

and the voltage regulation curve, for r=zero, is obtained by evaluating equations 9 and 13, taking corresponding values of σ and λ from fig. 27, and plotting them as shown in fig. 28, curve A.

λ	σ	$\frac{v}{E}$	$\frac{i}{i_{sc}}$ fig. 28	
			curve A	curve B
$\pi = 180°$	90°	0	1·0	1·0
$\frac{2\pi}{3} = 120$	73·5	0·284	0·442	0·6
$\frac{\pi}{3} = 60$	39·5	0·772	0·04	0·1
0	0	1·0	0	0

Fig. 28 shows the maximum value of $\frac{v}{E}$ to be unity, but as has been shown the ratio on open circuit, when i=zero, is $\frac{m}{\pi}\sin\frac{\pi}{m}$ and this only approaches unity as m becomes large enough to make $\sin\frac{\pi}{m}$ equal to $\frac{\pi}{m}$ radians. Curve A, therefore, is strictly true only for a large number of anodes.

For x=zero equation 10 becomes

$$i' = \frac{E}{r}(\cos\theta - \cos\sigma) \quad . \quad . \quad . \quad . \quad (14)$$

The curve connecting σ and λ, from no-load to full load, is determined from equation 14 using the terminal condition

$$i' = 0 \text{ when } \theta = 2\lambda - \sigma, \text{ fig. 25}$$

so that $$\cos(2\lambda - \sigma) = \cos\sigma$$

from which $$\lambda = \sigma$$

The connexion between λ and σ for x=zero is shown

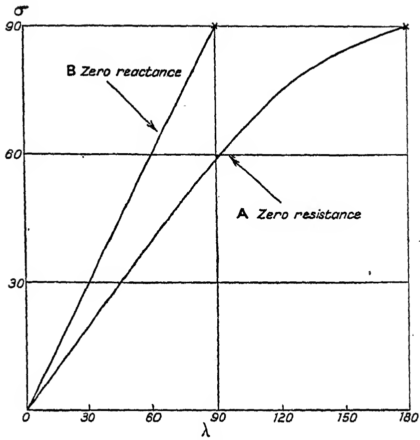

σ
90
60
30
B Zero reactance
A Zero resistance
0
30
60
90
120
150
180
λ

FIG. 27

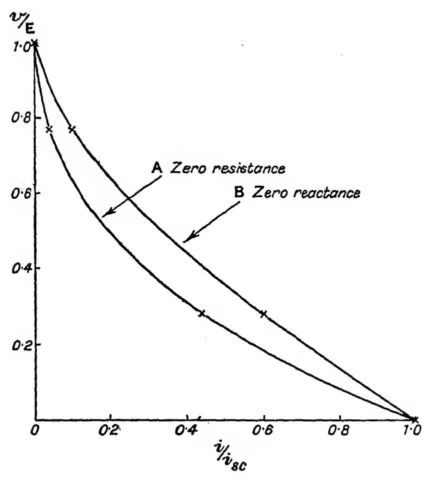

v/E
1·0
0·8
0·6
0·4
0·2
A Zero resistance
B Zero reactance
0
0·2
0·4
0·6
0·8
1·0
i/i_sc

FIG. 28

graphically by fig. 27, curve B, which is probably of the correct shape because, with zero reactance, the current flows from 0 to π and λ must increase at the same rate as σ, see fig. 26.

The mean current output from the anode, over the period 0–2π, is from equation 14

$$\frac{i}{m}=\frac{E}{2\pi r}\int_{-\sigma}^{+\sigma}[\cos\theta-\cos\sigma]d\theta$$

$$\therefore i=\frac{mE}{\pi r}[\sin\sigma-\sigma\cos\sigma]$$

and $$i_{sc}=\frac{mE}{\pi r} \text{ for a complete short circuit where}$$

$$\sigma=\frac{\pi}{2}$$

$$\therefore \frac{i}{i_{sc}}=[\sin\sigma-\sigma\cos\sigma]\quad . \quad . \quad . \quad . \quad (15)$$

and the voltage regulation curve for x=zero is obtained by evaluating equations 9 and 15, taking values of σ between zero and $\frac{\pi}{2}$, and plotting them as shown in fig. 28, curve B.

It should be noted that the actual magnitudes of the short-circuit currents i_{sc} are not the same in both cases considered and that if r in the one case is equal to x in the other case, then the short-circuit current $\frac{mE}{x}$ for a reactive circuit is 3·14 times as large as the current $\frac{mE}{\pi r}$ for the resistive circuit. For any given value of i, since i_{sc} can be calculated, the value of the ratio $\frac{i}{i_{sc}}$ is known and the corresponding value of $\frac{v}{E}$ is obtained from fig. 28.

CHAPTER VII

RECTIFICATION, INVERSION, AND REGENERATION

THE fact that a mercury arc can work inverted, that is, absorb D.C. energy and transfer it to the A.C. side, is well known. The anode-cathode circuits must possess inductance and the A.C. mains must be alive to fix the frequency, to supply the wattless kVA of the arc transformer and to extinguish the arc at the correct instants as required by commutation. The kVA supply of live A.C. need only be small, but it cannot be dispensed with.

Consider first a hexaphase mercury arc 'rectifying' and charging a battery in the same manner as is shown in fig. 29, and in particular consider the E.M.F. wave CDEFG.

If the positive grid bias is applied at the instant corresponding to the point C, fig. 29, the anode potential will follow the full line tips of the sine curves and have a high mean positive value relative to the transformer neutral and, when the arc drop is subtracted, the mean cathode positive potential relative to the transformer neutral is obtained, this being the mean value of the D.C. voltage charging the battery.

If commutation is retarded by causing the angle σ to increase through delaying the instant at which the positive impulse is given to the grid, the mean anode potential line will fall towards zero. Under circumstances such as obtain with incomplete rectification and neglecting the arc drop, the mean anode potential will fall to zero when the angle σ becomes 90 degrees as shown in figs. 14 and 29 where the anode potential fluctuates between D and E.

Since the mean value of the D.C. voltage falls continuously as the angle σ is increased it would be necessary to continuously reduce the back E.M.F. of the battery by gradually cutting down the number of cells, otherwise the battery would not charge.

Now consider the arc 'Inverting' with a given number of cells in the battery and σ increased beyond 90 degrees to get on the lower half of the curve CDEFG. The battery must be reversed so that the transformer secondary neutral point is connected to the positive pole and the cathode to the negative pole so that the battery can discharge current from the anode to the cathode. Under these conditions the cathode potential

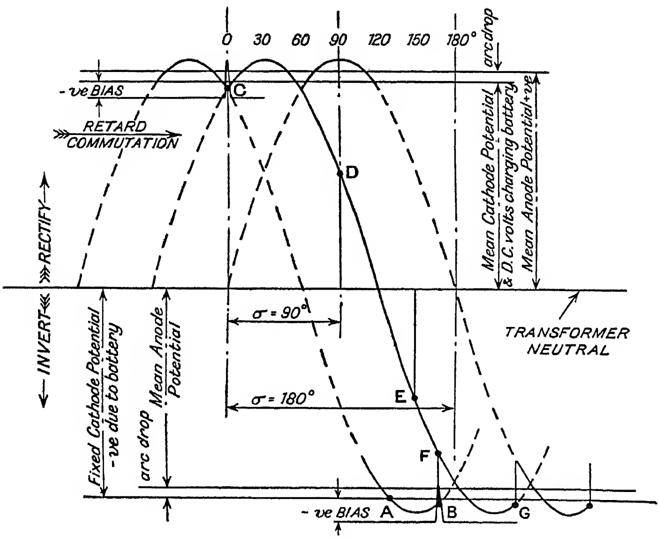


FIG. 29

will be held permanently at some fixed value below the transformer neutral point as shown in the lower half of fig. 29. Provided σ is large enough, the anode which has fired just in front of the one under consideration will definitely tend to become extinguished at the point A where its potential with respect to the cathode becomes negative. Inductance, however, will probably carry it on to some point such as B where it will finally be extinguished. At this point positive grid bias is applied to the anode under consideration which

fires at F and carries on to G where the next anode takes over. It will be noticed that the maximum value of σ must be less than 180 degrees measured from the original zero, and it is absolutely unsafe to retard σ to 180 degrees because any one anode must take over the arc while its potential is positive and higher, with respect to the cathode, than that of the preceding anode, otherwise all anodes will fire all the time and cause a total short circuit. For inverted working σ, in general, must be greater than $\frac{\pi}{2}$ but less than π, but where rectification is complete, σ may have to exceed $\frac{\pi}{2}+\frac{\pi}{m}$ (*see* fig. 14), and if this is so, then bi-phase arcs cannot operate inverted.

Owing to the short-circuit danger in the region $\sigma=\pi$, hard grid bias is better than soft grid bias for inverted operation. When short-circuit occurs, the curve EFG falls below the cathode potential line and stability is only regained by temporarily tripping the circuits. Once an arc is extinguished at a point such as G it remains extinguished because the grid then has a negative bias and it does not restrike until its grid is given a fresh positive bias at the appropriate point in the cycle.

The appropriate point at which to apply the positive bias is settled by the magnitude of the battery discharge voltage; the higher this voltage is, the greater is the retardation necessary, because excluding the arc drop the mean value of the anode voltage curve must be the cathode potential line. It is obvious from this that there is an upper limit to the battery discharge voltage which must be rather less than the maximum value of the sine wave of E.M.F. from the transformer secondary. It is also clear that any battery voltage less than the maximum can be dealt with provided that the angle σ is suitably chosen greater than 90 degrees but less than 180 degrees. In order to operate under these conditions it is advisable to commence with the angle σ greater than is actually required and to gradually reduce the angle until the desired discharge current flow is obtained. It will be seen from fig. 29 that, since the actual anode P.D. is fluctuating

with respect to the battery discharge voltage, it is essential to have a smoothing choke in the battery circuit between the cathode and the transformer secondary neutral point to prevent any violent current fluctuations. This poor D.C. wave shape is always present with retarded commutation both when rectifying and inverting.

That energy is being transferred from the D.C. side to the A.C. side under the inverted condition is readily appreciated when it is remembered that the current flow is always from the anode to the cathode but that, when inverted, the cathode is negative with respect to the transformer neutral point whereas it is positive when working as a rectifier and transferring energy from the A.C. side to the D.C. side. It is a fundamental fact that the direction of the energy transference reverses when, with a given direction of current flow, the P.D. is reversed.

The three main applications for the regenerative control possible with the mercury arc are:

(*a*) D.C. traction motors in conjunction with an A.C. transmission system.

(*b*) large D.C. rolling mill motors where the supply is A.C.

(*c*) large A.C. induction motors.

Nothing further need be specially said about division (*a*) since these motors are small enough to work from a single rectifier with a reversing switch to alter the connexions between the D.C. side and the rectifier when it is desired to return energy to the supply system as has been described in connexion with fig. 29 and battery charging and discharging.

Division (*b*) would probably require special treatment as shown in fig. 30, since it would be undesirable to perform switching operations on large plant.

In Fig. 30 A would have the usual negative grid bias complete with positive impulses and the angle σ retarded to a point rather less than 90 degrees, whilst B would have negative bias only and the angle σ rather less than 180 degrees. Under these conditions the D.C. motor could be speeded up from rest, drawing an increasing current supply from A, as σ approaches zero. To slow the motor down, all

that is necessary is to suppress the positive impulses to the grids of A and apply them to B and since the motor connexions to B are reversed, as compared with A, the stored energy of the motor will be returned to the line via B as the angle σ approaches 90 degrees. If in B the angle σ is reduced below 90 degrees, the motor will again speed up, but in a direction opposed to that previously obtaining, and this process of speeding up, slowing down and reversing will go on as long as the grids of A and B are properly biased and the angles σ correctly adjusted and no appreciable amount of energy is required to operate this 'master' control gear.

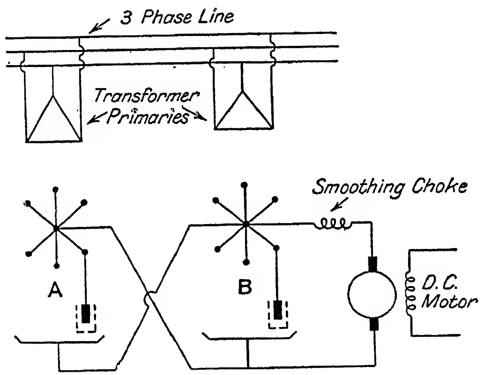


FIG. 30

Speed control in division (*c*) is characterized by the fact that since the rotor frequency is less than that of the stator, or supply, frequency it is necessary to be able to interconnect two systems of different frequency and this is accomplished as shown in fig. 31.

Note.—Two supplies of different frequency can be interconnected through two mercury arcs because one arc reduces the incoming frequency to zero and the other arc raises it to the outgoing frequency.

The essential difference between figs. 30 and 31 is that

whereas the transformer primaries in fig. 30 are on a common three-phase bus line, they are on separate bus lines in fig. 31. The operation of this system is similar to that of division (*b*), and if energy were absorbed from the rotor by B and returned to the supply by A, then the speed of the motor would be subsynchronous, whereas energy drawn from the supply through A and fed to the rotor through B would cause the speed to be supersynchronous much in

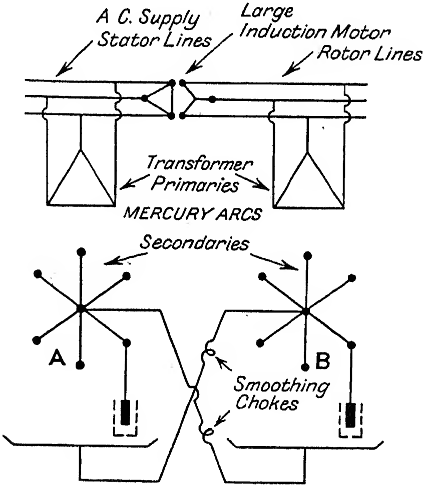


FIG. 31

the same manner as obtains when the 'Schrage' mechanism is fitted to induction motors for speed control. There are points of difference between the 'rectifier-invertor' mechanism and the 'Schrage' mechanism, the chief being that the 'Schrage' mechanism can pass the speed through synchronism, whereas the 'rectifier-invertor' mechanism cannot do this because at synchronous speed the frequency of the rotor currents is zero and the B transformer would fail to operate. The 'rectifier-invertor' mechanism, provided speed control

were confined to either subsynchronism or supersynchronism, would have a distinct advantage over the 'Schrage' mechanism for large motor operation owing to the fact that the 'Schrage' mechanism is limited in the voltage that can be handled by the commutator and, therefore, both in the amount of energy that can be transferred and in the A.C. voltage of the supply system. The amount of energy that can be handled by a 'rectifier-invertor' is very large since D.C. pressures of 50,000 volts and possibly 100,000 volts can be attained in one unit and, therefore, large speed ranges with large outputs are possible.

This interconnexion of two A.C. systems of different frequency can be applied to generating stations and the flow of energy between them is readily achieved by grid control, as has been described. The mercury arc also provides an easy method of obtaining high-tension D.C. for transmission purposes where this is desirable.

CHAPTER VIII

WAVE FORM ANALYSIS

FIG. 32 shows the no-load wave form for a single-phase, or single-anode, mercury arc and its composition can be readily ascertained by making use of the Fourier Theorem.

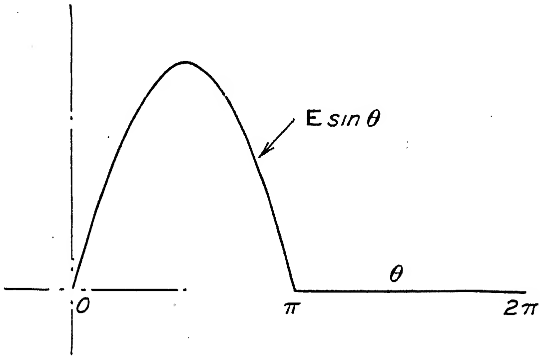


FIG. 32

According to this theorem, the expansion of $f(\theta)$ as a trigonometric series, periodic in 2π, is

$$f(\theta)=\frac{b_0}{2}+b_1\cos\theta+\ldots+b_n\cos n\theta+\ldots$$
$$+a_1\sin\theta+\ldots\infty$$

where n is the order of the harmonic under consideration and $\frac{b_0}{2}$ is the average value of the no-load E.M.F. over the period.

For fig. 32 in the range $0<\theta<\pi$, $f(\theta)=E \sin\theta$
and $\pi<\theta<2\pi$, $f(\theta)=0$
so that

$$a_n=\frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin n\theta.d\theta$$

$$=\frac{E}{\pi}\int_0^{\pi} \sin\theta \sin n\theta.d\theta$$

$$=\frac{E}{2\pi}\int_0^{\pi}[\cos (n-1)\theta-\cos (n+1)\theta]d\theta$$

$$=\frac{E}{\pi(n^2-1)}\Big[\sin n\theta \cos\theta-n \cos n\theta \sin\theta\Big]_0^{\pi}$$

$$=-\frac{E}{\pi(n^2-1)}[\sin n\pi+\sin n0]$$

$=$zero for all values of n except $n=1$

For $n=1$, a_n takes the form $\frac{0}{0}$ and the limiting value is obtained by differentiating both the numerator and denominator with respect to n and putting $n=1$, thus

$$a_1=\frac{E}{2} \text{ (limit } n=1)$$

Similarly

$$b_n=\frac{E}{\pi}\int_0^{\pi} \sin\theta \cos n\theta.d\theta$$

$$=\frac{E}{2\pi}\int_0^{\pi}[\sin (n+1)\theta-\sin (n-1)\theta]d\theta$$

$$=\frac{E}{\pi(n^2-1)}\Big[\cos n\theta \cos\theta+n \sin n\theta \sin\theta\Big]_0^{\pi}$$

$$=-\frac{E}{\pi(n^2-1)}[\cos n\pi+\cos n0]$$

so that b_n is zero for all odd values of n and gives, for all even values

$$b_n = -\frac{2E}{\pi(n^2-1)}$$

and

$$\frac{b_0}{2} = \frac{E}{\pi} \quad (n=0)$$

Thus, in the case of the single-phase arc, the equation to the no-load E.M.F. wave is found to be

$$e = E\left[\frac{1}{\pi} + \frac{\sin\theta}{2} - \frac{2\cos 2\theta}{3\pi} - \frac{2\cos 4\theta}{15\pi} - \ldots \infty\right] \quad (16)$$

and this shows that the no-load E.M.F. wave consists of a constant term together with a fundamental sine curve and a series of even cosine harmonics.

In the case of a bi-phase, or two-anode, mercury arc if the one-anode wave shape is represented by equation 16, then the other will be represented by a similar equation in which $\theta+\pi$ is substituted for θ and the total wave, which is the sum of the two parts, is

$$e = E\left[\frac{2}{\pi} - \frac{4\cos 2\theta}{3\pi} - \frac{4\cos 4\theta}{15\pi} - \ldots \infty\right] \quad . \quad (17)$$

Equation (17) is seen to consist of a constant term together with a series of even cosine harmonics.

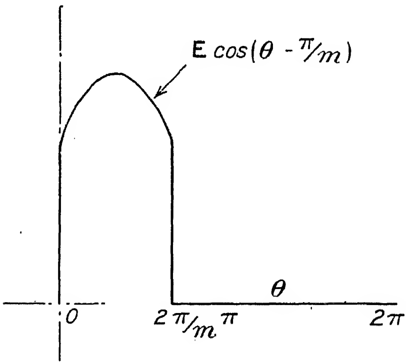


FIG. 33

In the two cases considered the discontinuity in the total E.M.F. curve occurs on the vertical axis through $\theta=0$, and if similar equations are required for each anode in the case of three and six anode arcs then it will be necessary to proceed as follows:

Fig. 33 shows the no-load E.M.F. wave form for any one anode of an 'm' phase mercury arc where $m>1$

For fig. 33 in the range

$$0<\theta<\frac{2\pi}{m},\ f(\theta)=E\cos\left(\theta-\frac{\pi}{m}\right)$$

and

$$\frac{2\pi}{m}<\theta<2\pi,\ f(\theta)=0$$

The expansion of $f(\theta)$ as a trigonometric series periodic in 2π is, as before

$$f(\theta)=\frac{b_0}{2}+b_1\cos\theta+\ldots+b_n\cos n\theta+\ldots$$
$$+a_1\sin\theta+\ldots\infty$$

so that

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f(\theta)\sin n\theta.d\theta$$

$$=\frac{E}{\pi}\int_0^{\frac{2\pi}{m}}\cos\left(\theta-\frac{\pi}{m}\right)\sin n\theta.d\theta$$

$$=-\frac{E}{\pi(n^2-1)}\left[\frac{n-1}{2}\cos(2n+1)\frac{\pi}{m}+\frac{n+1}{2}\cos(2n-1)\frac{\pi}{m}-n\cos\frac{\pi}{m}\right]$$

and

$$b_n=\frac{1}{\pi}\int_0^{2\pi}f(\theta)\cos n\theta.d\theta$$

$$=\frac{E}{\pi}\int_0^{\frac{2\pi}{m}}\cos\left(\theta-\frac{\pi}{m}\right)\cos n\theta.d\theta$$

$$=\frac{E}{\pi(n^2-1)}\left[\frac{n-1}{2}\sin(2n+1)\frac{\pi}{m}+\frac{n+1}{2}\sin(2n-1)\frac{\pi}{m}-\sin\frac{\pi}{m}\right]$$

For $n=1$, both a_n and b_n take the form $\frac{0}{0}$ and must be differentiated in the usual way. If, then, n is put equal to unity

$$a_1=\frac{E}{\pi}\sin\frac{\pi}{m}\left[\frac{\pi}{m}+\frac{1}{2}\sin\frac{2\pi}{m}\right]$$

$$b_1=\frac{E}{\pi}\cos\frac{\pi}{m}\left[\frac{\pi}{m}+\frac{1}{2}\sin\frac{2\pi}{m}\right]$$

$$\frac{b_0}{2}=\frac{E}{\pi}\sin\frac{\pi}{m}$$

From these results for each individual anode and in a manner similar to that adopted for the bi-phase arc, it can be shown that the total no-load E.M.F. wave for a tri-phase, or three-anode, arc is represented by the equation

$$e=E\left[\frac{3\sqrt{3}}{2\pi}-\frac{3\sqrt{3}\cos 3\theta}{8\pi}-\frac{3\sqrt{3}\cos 6\theta}{35\pi}-\ldots\infty\right] \quad . \quad (18)$$

and that for a hexaphase, or six-anode, arc by the equation

$$e=E\left[\frac{3}{\pi}-\frac{6\cos 6\theta}{35\pi}-\frac{6\cos 12\theta}{143\pi}-\ldots\infty\right] \quad . \quad . \quad . \quad . \quad (19)$$

From equation (19) it is seen that the hexaphase wave

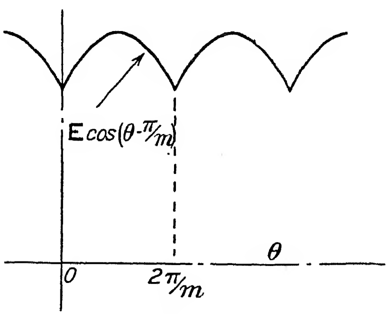


FIG. 34

consists substantially of a constant quantity with a superimposed sixth harmonic cosine term, and that the ratio

$$\frac{\text{Mean}}{\text{Maximum}}\ \text{Voltage}=\frac{3}{\pi}=0{\cdot}955$$

whilst the ratio

$$\frac{\text{R.M.S.}}{\text{Maximum}}\ \text{Voltage}=\sqrt{\left[\left(\frac{3}{\pi}\right)^2+\frac{1}{2}\left(\frac{6}{35\pi}\right)^2+\ldots\right]}=0{\cdot}956$$

which values agree with those previously obtained in Chapter III for the hexaphase arc.

The method outlined for determining the no-load E.M.F. wave shape due to all the anodes in an 'm' phase system, $m>1$, is rather troublesome to apply and, therefore, it is more convenient to deal with the no-load wave form as a whole thus:

Fig. 34 shows an 'm' phase wave form, the system being periodic in $\frac{2\pi}{m}$

In the range

$$0<\theta<\frac{2\pi}{m},\ f(\theta)=\text{E}\cos\left(\theta-\frac{\pi}{m}\right)$$

The expansion of $f(\theta)$ as a trigonometric series periodic in $\frac{2\pi}{m}$ is

$$f(\theta)=\frac{b_0}{2}+b_1\cos m\theta+\ldots+b_n\cos mn\theta+\ldots$$
$$+a_1\sin m\theta+\ldots\infty$$

then

$$a_n=\frac{m\text{E}}{\pi}\int_0^{\frac{2\pi}{m}}\cos\left(\theta-\frac{\pi}{m}\right)\sin mn\theta.d\theta$$
$$=0 \text{ for all values of } n$$

$$b_n=\frac{m\text{E}}{\pi}\int_0^{\frac{2\pi}{m}}\cos\left(\theta-\frac{\pi}{m}\right)\cos mn\theta.d\theta$$
$$=-\frac{2\text{E}}{m^2n^2-1}\left[\frac{m}{\pi}\sin\frac{\pi}{m}\right]$$

$$\frac{b_0}{2}=E\frac{m}{\pi}\sin\frac{\pi}{m}$$

and from these results equations 17, 18, and 19 are readily obtained.

Now in these equations $\frac{b_0}{2}$ is the mean value of the rectified E.M.F. wave, and b_n is the maximum value of the harmonic, so that the R.M.S. value of the harmonic is numerically equal to

$$\frac{\sqrt{2}E}{m^2n^2-1}\left[\frac{m}{\pi}\sin\frac{\pi}{m}\right]$$

and the ratio

$$\frac{\text{R.M.S. value harmonic}}{\text{Mean value rectified wave}}=\frac{\sqrt{2}}{m^2n^2-1}$$

from which the following table has been prepared.

Order of harmonic fundamental= 50		Harmonic R.M.S. value as percentage of rectified mean value			
		$m=2$	$m=3$	$m=6$	$m=12$
$mn=2$	100	47·2	—	—	—
3	150	—	17·7	—	—
4	200	9·44	—	—	—
6	300	4·05	4·05	4·05	—
8	400	2·25	—	—	—
9	450	—	1·77	—	—
10	500	1·43	—	—	—
12	600	0·99	0·99	0·99	0·99

From this table it is seen that, if a harmonic is present at all, then it is present as a definite percentage of the mean rectified value irrespective of the number of anodes employed. It is also apparent that the D.C. wave shape improves rapidly as the number of anodes increases.

CHAPTER IX

TRANSFORMER RATINGS

IN the design of mercury arc rectifying plant it is essential to be able to state the approximate relationship between the mean D.C. output from the arc and the kVA capacity of the transformer supplying it.

The kVA capacity of a transformer is usually understood to be its capacity when operating normally with a specified mean temperature rise, and a normal transformer is one which is both absorbing and delivering energy of an alternating character.

Although there are, at present, different views upon what is, or should be, implied by the term 'utility' factor as applied to the transformer of a rectifying plant, it appears obvious that the correct definition is the ratio of the actual mean D.C. output to the A.C. output as a normal transformer for the same mean temperature rise.

It will be apparent that the actual numerical value of the utility factor in any given case cannot be ascertained except as the direct result of experiment for the simple reason that the actual wave shapes when rectifying cannot be accurately predicted, nor are they capable of being expressed in simple mathematical form.

In the following theory sine laws have been assumed and it must be understood that the results are only true on the assumptions made and that other wave shapes, rectangular ones for instance, would have produced different values.

It has also been assumed that the power factor is unity and that the efficiency is 100 per cent, so that the transformer R.M.S. input and the arc R.M.S. output are equal. In practice this assumption is not, of course, true. The primary magnetizing current has also been neglected. Single Anode, single phase, Case:

As shown in fig. 35 in both primary and secondary windings for a ratio 1—1

$e = E \sin\theta$ from 0 to 2π

and $i = I \sin\theta$ from 0 to π and zero from π to 2π also the primary and secondary ampere turns balance.

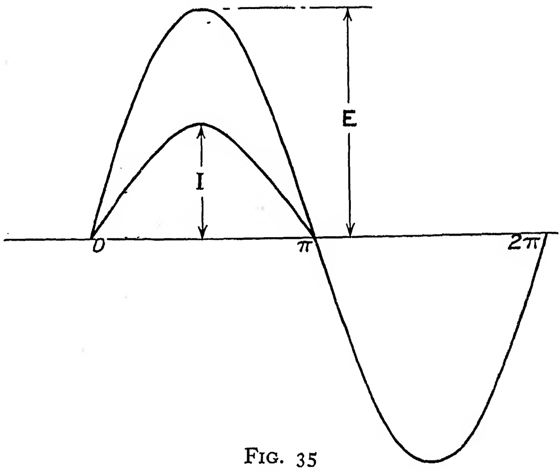


FIG. 35

If moving-coil type instruments were placed in the 'load' circuit they would read mean values of the output waves over the interval 0–2π which are

$$0{\cdot}318E \text{ and } 0{\cdot}318I \text{ (\textit{see} Chapter III)}$$

so that the mean D.C. watt output is $0{\cdot}101EI$.

If this transformer were operating as a normal single-phase transformer then, since the iron loss is not affected by the nature of the load, the same maximum voltage E can be applied to it.

The copper loss, however, and therefore the copper mean temperature rise depends upon the mean square value of the current over the whole period 0–2π so that, for equal copper temperature rises, the mean square values of the current over the period 0–2π must be equal.

In the case of the arc transformer this value is, since both primary and secondary are intermittently loaded,

$$\frac{I^2}{2\pi}\int_0^{\pi}\sin^2\theta.d\theta=\frac{I^2}{4}$$

If x=the maximum value of the current as a normal single-phase transformer the mean square value is, since both primary and secondary are continuously loaded,

$$\frac{x^2}{2\pi}\int_0^{2\pi}\sin^2\theta.d\theta=\frac{x^2}{2}$$

Thus for equal temperature rises

$$x=\frac{I}{\sqrt{2}}$$

and as a single-phase transformer operating under normal conditions and with the same mean temperature rise the R.M.S. A.C. output is

$$\frac{E}{\sqrt{2}}.\frac{I}{2}=0{\cdot}354EI \text{ volt-amperes}$$

so that the utility factor is $\frac{0{\cdot}101}{0{\cdot}354}=0{\cdot}285$.

Two-Anode, bi-phase, centre tapped secondary, Case:

Since R.M.S. inputs and outputs are equal the primary and secondary circuits are as shown in fig. 36, and it should be noticed that the primary and secondary ampere turns are equal.

If moving-coil type instruments were placed in the load circuit they would read mean values of the output waves which are

$$0{\cdot}636\times\frac{E}{2} \text{ and } 0{\cdot}636\times 2I$$

so that the mean D.C. watt output is $0{\cdot}404EI$.

In this case the primary loading is continuous, whilst that of the secondary sections is intermittent, and for equal mean

temperature rises in both primary and secondary windings it is necessary for them to have equal values for the ratio

$$\frac{\text{Total watt loss}}{\text{Total barrel coil surface}}$$

If the primary and secondary windings are arranged to have equal ratios of barrel coil surface to weight then they must have equal values for the ratio

$$\frac{\text{Total watt loss}}{\text{Weight copper}}$$

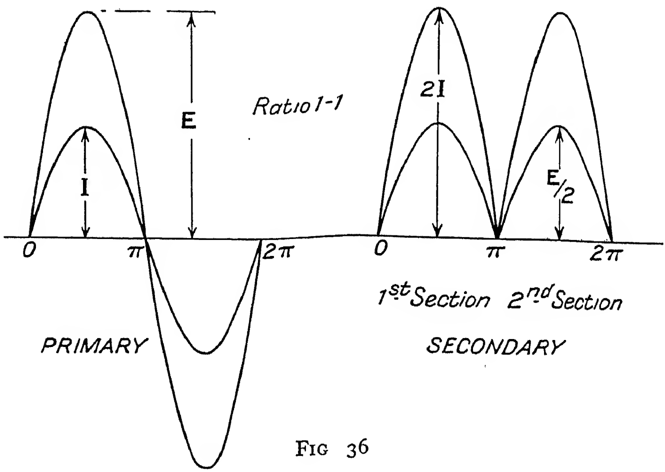


FIG 36

Now the watt loss in the copper in one section is proportional to the product of the weight of the copper and the mean square value of the current density in the section over the period o–2π, hence, for equal final mean temperature rises in both primary and secondary windings the mean square values of the current density in the windings from o to 2π must be equal so that

$$\left(\frac{I}{a}\right)^2 \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta.d\theta. = \left(\frac{2I}{a'}\right)^2 \frac{1}{2\pi}\int_0^{\pi} \sin^2\theta.d\theta$$

Where a and a' are the cross-sectional areas of the primary and secondary conductors respectively.

Thus $$a'=\sqrt{2}a \text{ and } r=\sqrt{2}r'$$

and let r and r' be the ohmic resistances of the complete primary and secondary windings. Therefore for equal final mean temperature rises in both windings the cross-sectional area of the secondary is 1·41 times its strictly equivalent primary value.

The total copper loss due to both primary and secondary windings is, when operating the arc,

$$\frac{rI^2}{2\pi}\int_0^{2\pi}\sin^2\theta.d\theta+\frac{r}{\sqrt{2}}\frac{(2I)^2}{2\pi}\int_0^{\pi}\sin^2\theta.d\theta$$
$$=1{\cdot}207rI^2$$

As a normal single-phase transformer whose maximum current, in both primary and secondary windings, is x amperes the total copper loss is

$$\frac{rx^2}{2\pi}\int_0^{2\pi}\sin^2\theta.d\theta+\frac{r}{\sqrt{2}}\frac{x^2}{2\pi}\int_0^{2\pi}\sin^2\theta.d\theta$$
$$=0{\cdot}854rx^2$$

If this condition is to give the same final mean temperature rise as before then since the weight of the copper is unaltered

$$0{\cdot}854x^2=1{\cdot}207I^2$$

so that $$x=1{\cdot}19I$$

The permissible output as a normal single-phase transformer is therefore

$$1{\cdot}19\frac{EI}{2}=0{\cdot}595EI \text{ volt-amperes}$$

so that the utility factor is

$$\frac{0{\cdot}404}{0{\cdot}595}=0{\cdot}68$$

Using the standard Bridge Connexion, instead of the centre tapped secondary, the transformer is of normal single-phase construction and there is no question of increasing the sectional area of the secondary conductors.

With sinusoidal wave shapes the mean D.C. output is

$$0{\cdot}636E \times 0{\cdot}636I = 0{\cdot}404EI$$

As a normal transformer the R.M.S. A.C. output is

$$\frac{EI}{2} = 0{\cdot}5EI$$

and the utility factor is $0{\cdot}404/0{\cdot}5 = 0{\cdot}808$.

A bi-phase arc has a tendency to produce a rectangular current wave which would have a mean value of $0{\cdot}707I$ for the same heating effect as a sinusoidal wave. Under these conditions the mean D.C. output is

$$0{\cdot}636E \times 0{\cdot}707I = 0{\cdot}45EI$$

and the utility factor is $\dfrac{0{\cdot}45}{0{\cdot}5} = 0{\cdot}9$

Three-Anode, three-phase, Case:

Note.—This particular scheme of connexions with a sinusoidal primary current is only possible when the transformer carries a tertiary winding.

In the primary circuit the R.M.S. input volt-amperes

$$= \frac{3EI}{2}$$

and the D.C. R.M.S. output volt-amperes

$$= 0{\cdot}84E \times 0{\cdot}84I'$$

so that the maximum value of the secondary current wave in one phase of the winding is, as shown in fig. 37

$$I' = 2{\cdot}12I$$

which value, however, does not permit of an exact balance between primary and secondary ampere turns.

Moving-coil instruments in the D.C. output circuit would read mean values of the output waves which are

$$0{\cdot}825E \text{ and } 0{\cdot}825 \times 2{\cdot}12I$$

so that the mean D.C. watt output is $1{\cdot}45EI$.

For equal final mean temperature rises in both primary

and secondary windings, as shown in the bi-phase case

$$\left(\frac{I}{a}\right)^2 \frac{1}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta = \left(\frac{2\cdot 12I}{a'}\right)^2 \frac{1}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin^2 \theta.d\theta$$

so that $a' = 1\cdot 45a$ and $r = 1\cdot 45r'$

where r and r' are the ohmic resistances of either of the phases.

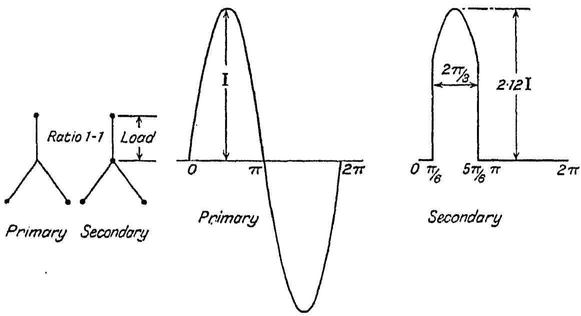


FIG. 37

The total copper loss due to both windings is, when operating the arc,

$$3\left[\frac{rI^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta + \frac{r}{1\cdot 45}\,\frac{(2\cdot 12I)^2}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin^2 \theta.d\theta\right]$$

$$= 3\cdot 68rI^2$$

As a normal three-phase transformer whose maximum phase current is x amperes the total copper loss is

$$3\left[\frac{rx^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta + \frac{r}{1\cdot 45}\,\frac{x^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta\right]$$

$$= 2\cdot 55rx^2$$

Thus for equal final mean temperature rises in both cases

$$2\cdot 55x^2 = 3\cdot 68I^2$$

so that $x = 1\cdot 205I$

and the output as a normal three-phase transformer is

$$3\left(\frac{E\times 1{\cdot}205I}{2}\right)=1{\cdot}81EI \text{ volt-amperes}$$

and the utility factor is

$$\frac{1{\cdot}45}{1{\cdot}81}=0{\cdot}8$$

Six-Anode, hexaphase, Case:

In the primary circuit the R.M.S. input volt-amperes

$$=\frac{3EI}{2}$$

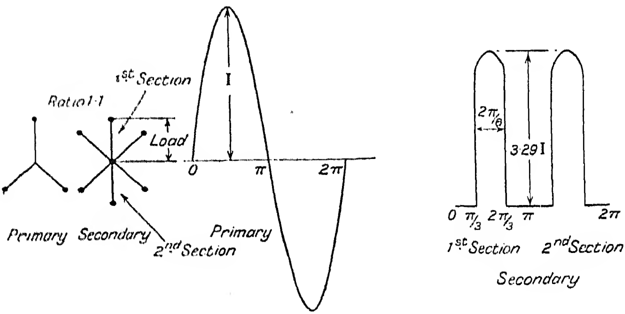


FIG. 38

and the D.C. R.M.S. output volt-amperes

$$=0{\cdot}956\frac{E}{2}\times 0{\cdot}956I'$$

so that the maximum value of the secondary current is, in the sections of the winding, as shown by fig. 38

$$I'=3{\cdot}29I$$

which produces a rather worse ampere-turn balance than the three-phase case.

Moving-coil instruments in the D.C. output circuit would read mean values of the output waves which are

$$0{\cdot}955\times\frac{E}{2} \text{ and } 0{\cdot}955\times 3{\cdot}29I$$

so that, for all practical purposes, the mean D.C. watt output is 1·5EI.

For equal final mean temperature rises in both primary and secondary windings as shown in the bi-phase case,

$$\left(\frac{I}{a}\right)^2 \frac{1}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta = \left(\frac{3\cdot29I}{a'}\right)^2 \frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sin^2 \theta.d\theta$$

so that $a' = 1\cdot81a$ and $r = 1\cdot81r'$

Let r = ohmic resistance of one primary phase

and r' = ohmic resistance of one whole secondary phase (two sections).

Then the copper loss due to both windings is, when operating the arc,

$$3\left[\frac{rI^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta + \frac{r}{1\cdot81}.\frac{(3\cdot29I)^2}{2\pi}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sin^2 \theta.d\theta\right]$$

$$= 4\cdot23rI^2$$

As a normal three-phase transformer whose maximum current is x amperes the total copper loss is

$$3\left[\frac{rx^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta + \frac{r}{1\cdot81}\,\frac{x^2}{2\pi}\int_0^{2\pi} \sin^2 \theta.d\theta\right]$$

$$= 2\cdot33rx^2$$

Thus for equal final mean temperature rises in both cases

$$2\cdot33x^2 = 4\cdot23I^2$$

so that

$$x = 1\cdot345I$$

and the output as a normal three-phase transformer is

$$3\left(\frac{E \times 1\cdot345I}{2}\right) = 2\cdot02EI \text{ volt-amperes}$$

and the utility factor is

$$\frac{1\cdot5}{2\cdot02} = 0\cdot743$$

These results have been plotted as fig. 39. Where

$$\text{Utility factor} = \frac{\text{Mean D.C. watt output}}{\text{Normal R.M.S. volt-amperes A.C.}}$$

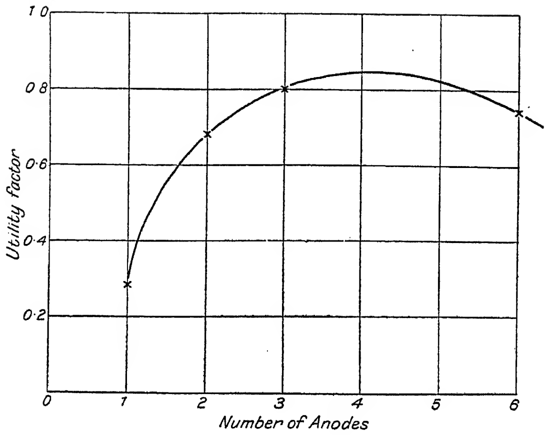

1 0
0 8
0·6
0·4
0·2
Utility factor
0
1
2
3
4
5
6
Number of Anodes

FIG. 39

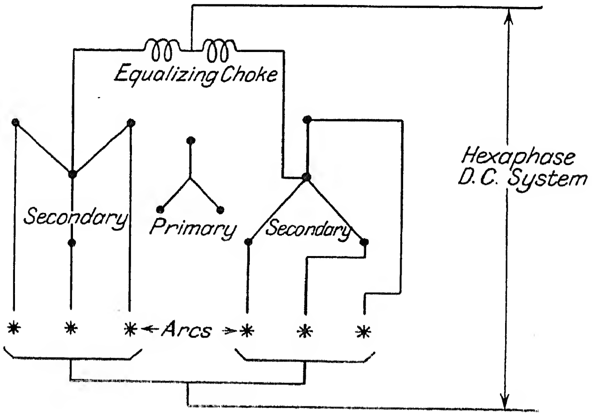

Equalizing Choke
Secondary
Primary
Secondary
Hexaphase
D.C. System
Arcs

FIG. 40

The fact that the curve shows the best utility factor to occur with three anodes probably led to the system of operating two three-anode arcs as shown in fig. 40 to give a six-anode output on the D.C. side.

In this double three-phase system the two sets of mercury arcs can either be in the same chamber or in separate chambers.

The results from which the curve fig. 39 has been plotted can be summarized as follows for any case except $m=1$.

Assuming for simplicity a ratio 1—1

Let r_1 be the resistance one whole phase of primary winding and a_1 its cross-sectional area.

r_2 be the resistance one whole phase of secondary winding and a_2 its cross-sectional area.

I_1=max. primary current as arc transformer.

I_2=max. secondary current as arc transformer.

x=max. primary and secondary current as normal A.C. transformer.

m=no. of anodes.

k=coefficient whose numerical value is equal to the number of phases for which transformer primary is wound.

If when operating as a rectifier both primary and secondary currents follow sine laws, as shown in figs. 36, 37, and 38,

then
$$\frac{I_2}{I_1}=\left[\frac{\int_0^{2\pi}\sin^2\theta.d\theta}{\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}+\frac{\pi}{m}}\sin^2\theta.d\theta}\right]=\frac{\phi}{\psi}$$

$$\frac{a_2}{a_1}=\frac{r_1}{r_2}=\sqrt{\frac{\phi}{\psi}}$$

Total copper loss as arc transformer

$$=k\frac{r_1 I_1^2}{2\pi}\left[\phi+\frac{\phi^2}{\sqrt{(\phi\psi)}}\right]$$

Total copper loss as normal A.C. transformer

$$=k\frac{r_1 x^2}{2\pi}[\phi+\sqrt{(\phi\psi)}]$$

$$x=\mathrm{I}_1\sqrt[4]{\frac{\phi}{\psi}}$$

$$\text{Utility factor}=\frac{\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right)^2\frac{\phi}{\psi}}{\frac{m}{2}\sqrt[4]{\frac{\phi}{\psi}}}$$

The following table gives the numerical values of the quantities involved:

k	m	ϕ	ψ	$\frac{\phi}{\psi}$	$\sqrt{\frac{\phi}{\psi}}$	$\sqrt[4]{\frac{\phi}{\psi}}$	$\left(\frac{m}{\pi}\sin\frac{\pi}{m}\right)^2$	Utility factor
1	2	3·14	1·57	2·0	1·41	1·19	0·405	0·68
3	3	3·14	1·48	2·12	1·45	1·205	0·68	0·8
3	6	3·14	0·956	3·29	1·81	1·345	0·912	0·74

It should be observed that these values of the utility factor should be regarded as comparative rather than actual since actual values could only be obtained experimentally. These values, however, serve to show the kind of thing to be expected in practice.

CHAPTER X

POWER FACTOR

SPECIAL points in the theory underlying the estimation of the power factor of a mercury arc installation are as follows: In the ordinary case of single-phase A.C. circuits, where the applied line E.M.F. produces a line current wave shape similar to itself, the power factor is numerically equal to the cosine of the angular displacement of the two waves concerned.

In the case of three-phase circuits the power factor is not numerically equal to the cosine of the angular displacement of the line voltage and current waves but is, in a balanced system, equal to the cosine of the angular displacement of the phase voltage and current waves. Three phase systems are, however, often unbalanced which rules out the phase displacement idea of power factor.

In the case of a mercury arc installation no matter for how many phases it is built, the wave shape of the current always differs from that of the E.M.F., and this shape difference produces a 'power' factor quite irrespective of whether there is any phase difference or not. If there is any difference of phase, or angular displacement of the waves, then it serves to accentuate the deviation of the power factor from unity.

Then, again, the arc itself is really a commutator, and its function is to open and close circuits which are inductive and, as has been shown in Chapter IV, the total current which flows under these conditions is not simply the permanent current but this permanent current with a superimposed transient. This superimposition of the transient on the permanent current not only causes the total current to be more nearly in phase with the E.M.F. but it also raises its R.M.S. value.

Thus in the case of a mercury arc installation it is evident

that the power factor is higher than might be imagined at first sight. From a practical point of view it is difficult to assign any definite value to the power factor owing to the irregular shape of the current waves.

Fig. 17, Chapter IV, shows a case of switching on an inductive circuit in which the permanent current lags by

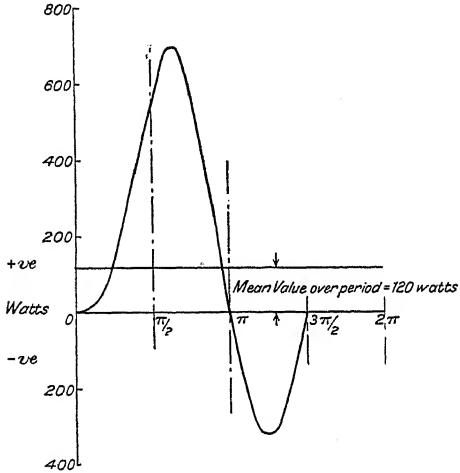


FIG. 41

72 degrees, giving a permanent power factor of 0·31. The R.M.S. voltage is 70·7, the R.M.S. permanent current is 4·47 amperes, and the resistance of the circuit is 5 ohms.

Over one complete period, however, and taking the transient current into account, the product of the E.M.F. and total current waves is as shown in fig. 41, the mean power over the period being 120 watts. The R.M.S. value of the total current is estimated to be 4·85 amperes, so that the initial value of the power factor is 0·35, which is considerably higher than the permanent value.

Further, it has been pointed out by several writers, notably Steinmetz (*see* Bibliography), that the ohmic resist-

ance of an electric arc, when carrying a sinusoidal current $i=\mathrm{I}\sin\theta$, is represented by the equation

$$r=\mathrm{R}(1+\varepsilon\cos 2\theta) \text{ where } \varepsilon<1{\cdot}0$$

There is, therefore, a double frequency oscillation of the resistance about a mean value R, the high value corresponding to a low current and the low value to a high current. The instantaneous power in the arc is

$$\begin{aligned} ri^2&=\mathrm{RI}^2\sin^2\theta(1+\varepsilon\cos 2\theta)\\ &=\mathrm{RI}^2[\sin^2\theta+\varepsilon\sin^2\theta-2\varepsilon\sin^4\theta] \end{aligned}$$

the mean power is

$$\begin{aligned} &\mathrm{RI}^2\frac{1}{2\pi}\int_0^{2\pi}[\sin^2\theta+\varepsilon\sin^2\theta-2\varepsilon\sin^4\theta]d\theta\\ &=\frac{\mathrm{RI}^2}{2\pi}\left[\pi+\varepsilon\pi-\frac{3}{2}\varepsilon\pi\right]\\ &=\frac{\mathrm{RI}^2}{2}\left[1-\frac{\varepsilon}{2}\right] \end{aligned}$$

Now the instantaneous voltage is

$$e=ri=\mathrm{RI}\sin\theta\,[1+\varepsilon\cos 2\theta]$$

So that the R.M.S. value of the voltage is

$$\begin{aligned} &\mathrm{RI}\sqrt{\left[\frac{1}{2\pi}\int_0^{2\pi}(\sin\theta+\varepsilon\sin\theta.\cos 2\theta)^2d\theta\right]}\\ &=\frac{\mathrm{RI}}{\sqrt{2}}\sqrt{\left[1-\varepsilon+\frac{\varepsilon^2}{2}\right]} \end{aligned}$$

and since the R.M.S. value of the current $=\dfrac{\mathrm{I}}{\sqrt{2}}$

the R.M.S. volt-amperes $=\dfrac{\mathrm{RI}^2}{2}\sqrt{\left[1-\varepsilon+\dfrac{\varepsilon^2}{2}\right]}$

and the power factor, that is the ratio mean power/R.M.S. volt-amperes

$$=\frac{1-\dfrac{\varepsilon}{2}}{\sqrt{\left[1-\varepsilon+\dfrac{\varepsilon^2}{2}\right]}}$$

which quantity is less than unity.

Thus a circuit containing pure resistance may have an 'inherent power' factor without any ordinary phase displacement between the applied voltage and the current flowing.

To consider this matter a little further assume a sinusoidal voltage wave

$$e = E_1 \sin \theta$$

producing, in a non-inductive circuit, a complex current wave

$$i = I + I_1 \sin \theta + I_2 \sin 2\theta + \ . \ . \ .$$

Then the mean power, as measured by a wattmeter is

$$\frac{1}{2\pi}\int_0^{2\pi} ei.d\theta$$

$$= \frac{E_1 I_1}{2}$$

The R.M.S. voltage, measured by a voltmeter $= \frac{E_1}{\sqrt{2}}$ and the R.M.S. current, measured by an ammeter

$$= \sqrt{[I^2 + \tfrac{1}{2}(I^2{}_1 + \ . \ . \ .)]}$$

so that the ratio $\dfrac{\text{Mean Power}}{\text{R.M.S. volt-amperes}}$

$$= \frac{\frac{I_1}{\sqrt{2}}}{\sqrt{[I^2 + \frac{1}{2}(I^2{}_1 + \ . \ . \ .)]}} = \mu$$

and μ is less than unity as is always the case when the wave shape of the current differs from that of the voltage. It will be noticed that μ is the value of the ratio

$$\frac{\text{Fundamental R.M.S. value}}{\text{Complex R.M.S. value}}$$

Now assume a sinusoidal voltage wave

$$e = E_1 \sin \theta$$

producing, in an inductive circuit, a complex current wave

$$i = I + I_1 \sin(\theta - \phi) + I_2 \sin(2\theta - \psi) + \ . \ . \ .$$

In this case the mean power is

$$\frac{1}{2\pi}\int_0^{2\pi} ei.d\theta=\frac{E_1I_1}{2}\cos\phi$$

The R.M.S. voltage$=\dfrac{E_1}{\sqrt{2}}$

and the R.M.S. current$=\sqrt{[I^2+\frac{1}{2}(I^2_1+\ldots)]}$
So that the ratio

$$\frac{\text{Mean Power}}{\text{R.M.S. volt-amperes}}=\frac{\dfrac{I_1}{\sqrt{2}}}{\sqrt{[I^2+\frac{1}{2}(I^2_1+\ldots)]}}\cos\phi$$

$$=\mu\cos\phi$$
$$=\lambda$$

μ is usually termed the 'distortion' factor, $\cos\phi$ the 'displacement' factor, and λ the 'inherent' power factor.

The 'distortion' factor being the ratio of the R.M.S. value of the fundamental component to that of the total wave, the 'displacement' factor being the cosine of the phase angle between the voltage wave and the fundamental component of the current wave and the 'inherent' power factor the ratio of the mean power to the R.M.S. volt-amperes.

In any actual installation since

$$\lambda=\frac{\text{Mean Power}}{\text{R.M.S. volt-amperes}}$$

and since, where the two-wattmeter method can be used for measuring the mean power,

$$\tan\phi=\sqrt{3}\frac{W_1-W_2}{W_1+W_2}$$

Where W_1 and W_2 are the wattmeter readings on a balanced load it is easy to determine $\cos\phi$ and consequently μ.

From what has been said it is apparent that no very precise value of the power factor can be given because it is impossible to state the exact shape of the current waves,

but if definite wave shapes are assumed then the estimation is fairly simple.

In a hexaphase rectifier assume that fig. 42 (*a*) represents the secondary rectified current wave shape for a pair of diametral anodes over the period 0–2π.

Since the sum of the transformer primary phase currents must always be zero, therefore (*b*) might well represent these phase currents and the transformer primary line current will be similar either to (*c*) for group 1 connexions or to (*d*) for group 2 connexions.

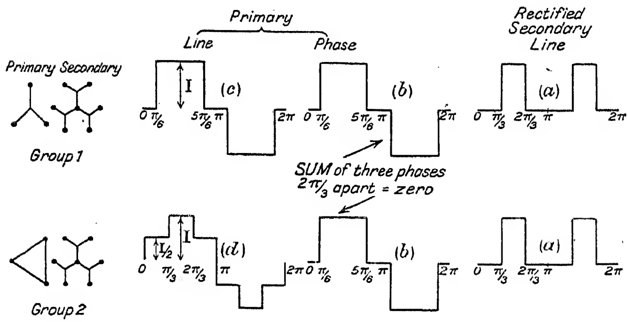


FIG. 42

Since both (*c*) and (*d*) are symmetrical with respect to axes through $\frac{\pi}{2}$ and π, by the Fourier Theorem they contain no constant term, no cosine terms and no even sine terms. The truth of this statement is most easily demonstrated by sketching a fundamental sine curve and then adding harmonics of various kinds and observing the shape of the resultant curve.

The odd sine terms are given by the equation

$$a_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} i \sin n\theta . d\theta$$

Thus for the primary line current (d), group 2,

$$a_n = \frac{4I}{\pi}\left[\frac{1}{2}\int_0^{\frac{\pi}{3}} \sin n\theta.d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\ n\theta.d\theta\right]$$

$$= \frac{4I}{n\pi}\left[0{\cdot}5 + 0{\cdot}5 \cos\frac{n\pi}{3}\right]$$

Evaluating this expression and noting that for

$n = 3$, $a_n = 0$

the R.M.S. value of the fundamental $= 0{\cdot}675I$
and the R.M.S. value of the complex wave $= 0{\cdot}707I$
so that the distortion factor $\mu = 0{\cdot}96$

but the value of λ will be less than this in a practical case owing to the magnetizing current even though the secondary circuits be non-inductive.

For the primary line current (c), group 1,

$$a_n = \frac{4I}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin n\theta.d\theta$$

$$= \frac{4I}{n\pi}\cos\frac{n\pi}{6}$$

Here also for $n = 3$, $a_n = 0$

the R.M.S. value of the fundamental $= 0{\cdot}788I$
and the R.M.S. value of the complex wave $= 0{\cdot}816I$
so that the distortion factor $\mu = 0{\cdot}96$ as before.

If it is preferred to calculate the distortion factor from phase curves (b) rather than line curves, then the (c), group 1, calculation applies to both star and mesh connexions.

It is probable, therefore, that the scheme of connexions employed, star or mesh primary windings, has little effect on the power factor and that the value of the power factor is fairly constant above any current value large enough to mask the effect of the primary magnetizing current.

At light loads the influence of the magnetizing current is to reduce the power factor as shown in fig. 43.

In a tri-phase, 3-anode, rectifier assume that fig. 44 (*a*) represents the secondary rectified current wave shape for any one anode over the period 0–2π.

The transformer primary phase currents can be represented by (*b*) and the transformer primary line currents by (*c*) for group 1 and by (*d*) for group 2.

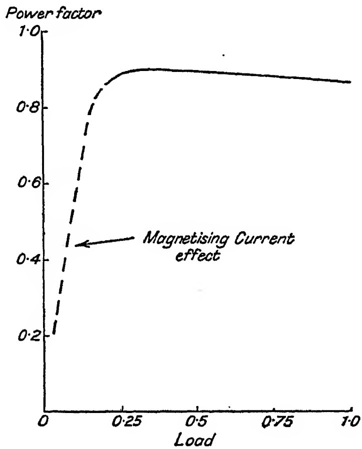


FIG. 43

Curve (*c*) contains sine terms only of all frequencies except those which are a multiple of 3 and for (*c*)

$$a_n = \frac{I}{\pi}\left[\int_0^{\frac{2\pi}{3}} \sin n\theta . d\theta - \int_{\frac{4\pi}{3}}^{2\pi} \sin n\theta . d\theta\right]$$

$$= \frac{3I}{n\pi}$$

whilst for (*d*), which contains sine and cosine terms of all frequencies except those which are a multiple of 3

$$a_n = \frac{2 \cdot 25 I}{n\pi} \quad \text{and} \quad b_n = \pm\frac{1 \cdot 3 I}{n\pi}$$

For both curves (c) and (d) the distortion factor $\mu = 0{\cdot}83$, and it is obvious that a consideration of the phase curves (b) will give the same value for μ.

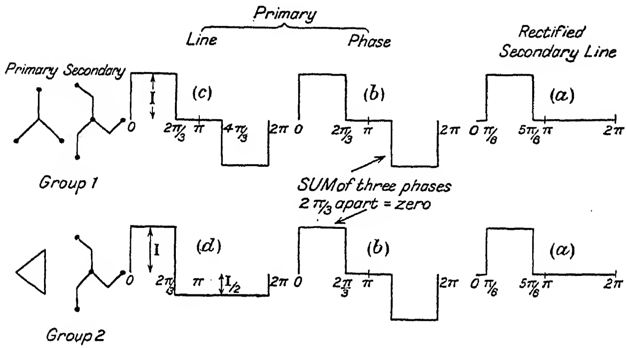


FIG. 44

Oscillograph records bear out the general shape of the line current waves in both figs. 42 and 44, although, naturally, the actual shapes are not as rectangular as those shown.

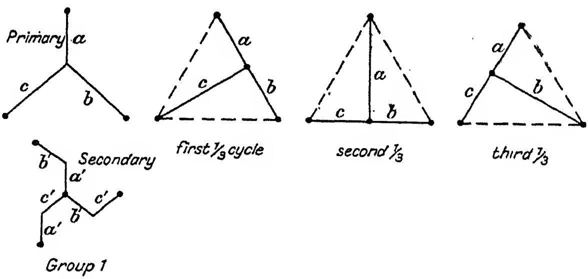


FIG. 45

The explanation of wave shape (c), fig. 44, is that, as shown in fig. 45, during the first third of the cycle the arc is carried by the secondary phases $a'b'$, which reduces the impedance of the primary phases a and b and leaves that of c high.

During the second third of the cycle $b'c'$ carry the arc, so that b and c are low and a is high. During the third part of the cycle c and a are low and b is high.

Thus the line current to the primary phase a is high during two-thirds of the complete cycle and low during one-third. The low value of the current is very little greater than the normal no-load current of the transformer in spite of the fact that, during the interval, the phase P.D. is high.

Fig. 45 shows the neutral point 'slipped' to the extreme position, a position which is probably not quite reached in practice.

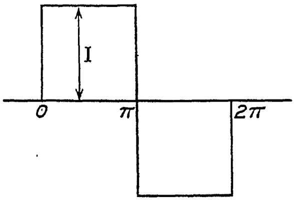


FIG. 46

When a tertiary winding is included on the transformer the oscillograph shows that the line current (c), fig. 44, is improved to a fairly good sine wave and demonstrates the value of a tertiary winding as a means of preventing distortion on the supply lines to a mercury arc.

In the case of a bi-phase rectifier, where the primary line current is assumed to be of the shape shown in fig. 46,

$$a_n = \frac{4I}{n\pi}$$

the R.M.S. value of the fundamental$=0{\cdot}9$I
and the R.M.S. value of the complex wave$=$I
so that $\mu = 0{\cdot}9$
which is higher than that for a tri-phase rectifier and lower than that for the hexaphase case.

CHAPTER XI

MATHEMATICS OF MERCURY ARCS

IN the following simplified mathematical treatment of the mercury arc the impedance of the primary side of the transformer is neglected and three standard types of differential equations are formed which are here appended together with their solutions.

In forming the equations it is assumed that during commutation two arcs burn, but not more than two, and that in the intervals between commutation only one arc burns.

It is also assumed that there is no delay in the firing of the arcs due to grid control.

Differential equations:

$$\text{1st type} \quad ri+x\frac{di}{d\theta}=-e$$

and the solution is $$i=A\varepsilon^{-a\theta}-\frac{e}{r}$$

In the transient term, A is a constant which depends upon the terminal conditions of the circuit and $a=\dfrac{r}{x}$

$$\text{2nd type} \quad \pm\left(ri+x\frac{di}{d\theta}\right)=E\sin\theta$$

and the solution is $$i=A\varepsilon^{-a\theta}\pm\frac{E}{z}\sin(\theta-\alpha)$$

The solution to this 2nd type always contains a permanent term, which is the ordinary current in an A.C. circuit lagging behind the applied voltage by some angle α and where

$$z=(r^2+x^2)^{\frac{1}{2}} \text{ and } \alpha=\tan^{-1}\frac{x}{r}$$

The transient term in the solution to the 2nd type is obtained by writing

$$ri+x\frac{di}{d\theta}=0$$

and must be similar to that in the 1st type.

$$\text{3rd type} \quad ri+x\frac{di}{d\theta}+e=\mathrm{E}\sin\theta$$

this is a combination of the 1st and 2nd types,

and the solution is $\quad i=\mathrm{A}\varepsilon^{-a\theta}-\frac{e}{r}+\frac{\mathrm{E}}{z}\sin(\theta-\alpha)$

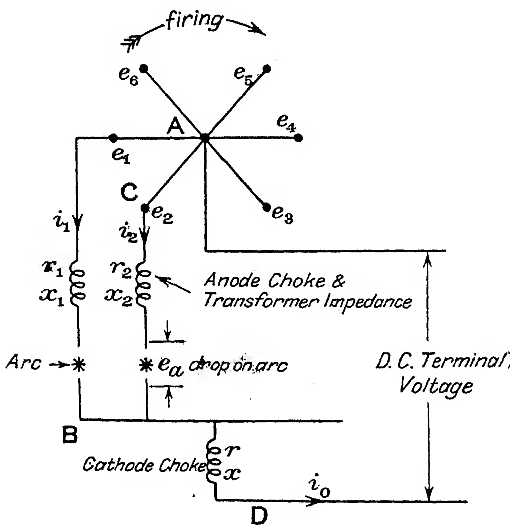


FIG. 47

The equations which follow have been formed specially for 2 and 6 anodes.

Figs. 47 and 48, which have been drawn to illustrate the

formation of the equations, represent a hexaphase, or six-anode, mercury arc.

In fig. 47 let

$i_0 = i_1 + i_2 =$ current during commutation
$R = r + r_1 \quad (r_1 = r_2 = r_3$ &c.)
$X = x + x_1 \quad (x_1 = x_2 = x_3$ &c.)
$z_1 = (r_1{}^2 + x_1{}^2)^{\frac{1}{2}}$
$Z = (R^2 + X^2)^{\frac{1}{2}}$
$Z_1 = \{(2R - r_1)^2 + (2X - x_1)^2\}^{\frac{1}{2}} = 2Z$ approx.
$e = e_a +$ D.C. terminal voltage.

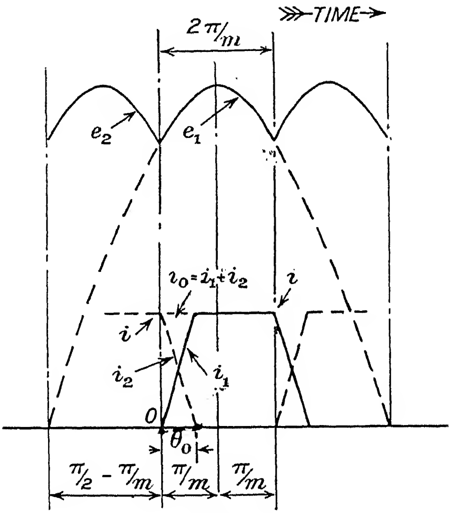


FIG. 48

Terminal conditions as depicted by fig. 48.

$i =$ mean value of rectified current and is assumed to

$$\text{occur at } \theta = 0 \text{ or } \theta = \frac{2\pi}{m}$$

Note.—This mean current value must occur at some

values of θ and it is most convenient to choose those named.

$$i_1(\theta=0)=0$$

$$i_0(\theta=0)=i_1\left(\theta=\frac{2\pi}{m}\right)=i$$

$$i_1(\theta=\theta_0)=i_0(\theta=\theta_0)$$

Also

$i_0=i_1+i_2$

θ_0=angle of overlap or period of commutation during which the arc is transferred from one anode to the next.

$$e_1=\text{E}\sin\left\{\left(\theta+\frac{\pi}{2}\right)-\frac{\pi}{m}\right\}=\text{E}\left(\cos\frac{\pi}{m}\cos\theta+\sin\frac{\pi}{m}\sin\theta\right)$$

$$e_2=\text{E}\sin\left\{\left(\theta+\frac{\pi}{2}\right)+\frac{\pi}{m}\right\}=\text{E}\left(\cos\frac{\pi}{m}\cos\theta-\sin\frac{\pi}{m}\sin\theta\right)$$

$$e_1-e_2=2\text{E}\left(\sin\frac{\pi}{m}\sin\theta\right)$$

Two arcs burning:

In the interval from $\theta=0$ to $\theta=\theta_0$ when two arcs are burning and going round the circuit ABC

$$e_1-e_2=2\text{E}\left(\sin\frac{\pi}{m}\sin\theta\right)=r_1i_1+x_1\frac{di_1}{d\theta}-r_1i_2-x_1\frac{di_2}{d\theta} \quad . \quad (20)$$

going round the circuit ABD

$$e_1=\text{E}\left(\cos\frac{\pi}{m}\cos\theta+\sin\frac{\pi}{m}\sin\theta\right)$$

$$=r_1i_1+x_1\frac{di_1}{d\theta}+ri_0+x\frac{di_0}{d\theta}+e \quad . \quad . \quad . \quad (21)$$

From equation (20) and since $i_0=i_1+i_2$

$$2\text{E}\left(\sin\frac{\pi}{m}\sin\theta\right)=-\left\{r_1(i_0-2i_1)+x_1\frac{d}{d\theta}(i_0-2i_1)\right\} \quad . \quad . \quad (22)$$

Since $\sin\dfrac{\pi}{m}$ is a constant this equation is of the 2nd type with the solution

$$(i_0-2i_1)=\mathrm{A}\varepsilon^{-a\theta}-2\sin\frac{\pi}{m}\frac{\mathrm{E}}{z_1}\sin(\theta-\alpha) \quad . \qquad \mathrm{I}$$

where $a=\dfrac{r_1}{x_1}$, $\alpha=\tan^{-1}\dfrac{x_1}{r_1}$ and the value of A, dependent upon the terminal conditions, is got by putting $\theta=0$, so that $i_1=0$ and $i_0=i$ whence

$$\mathrm{A}=i-2\sin\frac{\pi}{m}\frac{\mathrm{E}}{z_1}\sin\alpha$$

From equations (20) and (21), e_1+e_2 is written

$$2\mathrm{E}\left(\cos\frac{\pi}{m}\cos\theta\right)=(2\mathrm{R}-r_1)i_0+(2\mathrm{X}-x_1)\frac{di_0}{d\theta}+2e \qquad (23)$$

Since $\cos\dfrac{\pi}{m}$ is a constant this equation is of the 3rd type with the solution

$$i_0=\mathrm{B}\varepsilon^{-b\theta}-\frac{2e}{2\mathrm{R}-r_1}+2\cos\frac{\pi}{m}\frac{\mathrm{E}}{\mathrm{Z}}\cos(\theta-\beta) \quad . \qquad \mathrm{II}$$

where $$b=\frac{2\mathrm{R}-r_1}{2\mathrm{X}-x_1},\ \beta=\tan^{-1}\frac{2\mathrm{X}-x_1}{2\mathrm{R}-r_1}$$

and the value of B dependent upon the terminal conditions, is got by putting $\theta=0$ so that $i_0=i$ whence

$$\mathrm{B}=i+\frac{2e}{2\mathrm{R}-r_1}-2\cos\frac{\pi}{m}\frac{\mathrm{E}}{\mathrm{Z}}\cos\beta$$

One arc burning:

In the interval from $\theta=\theta_0$ to $\theta=\dfrac{2\pi}{m}$ when one arc only burns

$$e_1=\mathrm{E}\left(\cos\frac{\pi}{m}\cos\theta+\sin\frac{\pi}{m}\sin\theta\right)=\mathrm{R}i_1+\mathrm{X}\frac{di_1}{d\theta}+e \qquad (24)$$

$$\therefore\ \mathrm{E}\cos\left(\theta-\frac{\pi}{m}\right)=\mathrm{R}i_1+\mathrm{X}\frac{di_1}{d\theta}+e$$

this equation is of the 3rd type with the solution

$$i_1 = G\varepsilon^{-c\theta} - \frac{e}{R} + \frac{E}{Z}\cos\left(\theta - \frac{\pi}{m} - \gamma\right) \quad . \quad . \quad \text{III}$$

where
$$c = \frac{R}{X}, \ \gamma = \tan^{-1}\frac{X}{R}$$

and the value of G, dependent upon the terminal conditions, is got by putting $\theta = \frac{2\pi}{m}$ so that $i_1 = i$ whence

$$G = \varepsilon^{\frac{2\pi c}{m}}\left\{i + \frac{e}{R} - \frac{E}{Z}\cos\left(\frac{\pi}{m} - \gamma\right)\right\}$$

If the required values of e and i on the D.C. side are known as well as the resistances and reactances of the various circuits, then equation II can be used for the purpose of plotting the shape of the D.C. wave in the interval during which commutation takes place, that is, from $\theta = 0$ to $\theta = \theta_0$, whilst III can be used for the interval when one arc burns alone from $\theta = \theta_0$ to $\theta = \frac{2\pi}{m}$, but before the curve can be plotted it is necessary to be able to estimate E, the maximum value of the E.M.F. on any one of the transformer secondary coils, as well as the magnitude of the overlap angle θ_0.

Approximate equations for E and θ_0:

In equations I, II and III assume that the resistance terms are negligible compared with the reactance terms so that the angles of lag α, β, and γ become equal to $\frac{\pi}{2}$ and $a = b = c =$ zero, and since one of the terminal conditions is that when $\theta = \theta_0$ then $i_1 = i_0$, then under these conditions equation I becomes

$$i_1 = -A - 2\sin\frac{\pi}{m}\frac{E}{z_1}\cos\theta_0$$

also
$$A = i - 2\sin\frac{\pi}{m}\frac{E}{z_1}$$

$$\therefore\ i_1 = -i + 2\sin\frac{\pi}{m}\frac{E}{z_1}(1 - \cos\theta_0) \quad . \quad . \quad \text{I}a$$

Equation II becomes, by substituting $\mathcal{Z}=2Z$

$$i_1=B-\frac{2e}{2R-r_1}+\cos\frac{\pi}{m}\frac{E}{Z}\sin\theta_0$$

also
$$B=i+\frac{2e}{2R-r_1}$$

$$\therefore\ i_1=i+\cos\frac{\pi}{m}\frac{E}{Z}\sin\theta_0 \quad . \quad . \quad . \quad . \quad . \qquad \text{II}a$$

Equation III becomes

$$i_1=G-\frac{e}{R}+\frac{E}{Z}\sin\left(\theta_0-\frac{\pi}{m}\right)$$

also
$$G=\varepsilon^{\frac{2\pi c}{m}}\left(i+\frac{e}{R}-\frac{E}{Z}\sin\frac{\pi}{m}\right)$$

$$\therefore\ i_1=\varepsilon^{\frac{2\pi c}{m}}\left(i+\frac{e}{R}-\frac{E}{Z}\sin\frac{\pi}{m}\right)-\frac{e}{R}+\frac{E}{Z}\sin\left(\theta_0-\frac{\pi}{m}\right). \quad . \qquad \text{III}a$$

Equating Ia and IIa

$$E=\frac{i}{\dfrac{\sin\dfrac{\pi}{m}}{z_1}(1-\cos\theta_0)-\dfrac{\cos\dfrac{\pi}{m}}{2Z}\sin\theta_0} \quad . \quad . \qquad \text{IV}$$

It will be noticed that the second term in the denominator of equation IV becomes zero for the special case of a bi-phase arc where $m=2$, and this term is often neglected for all cases.

Equating IIa and IIIa and expanding the term

$$\frac{E}{Z}\sin\left(\theta_0-\frac{\pi}{m}\right)$$

$$\sin\frac{\pi}{m}\,\frac{E}{Z}\left(\varepsilon^{\frac{2\pi c}{m}}+\cos\theta_0\right)=\left(\varepsilon^{\frac{2\pi c}{m}}-1\right)\left(i+\frac{e}{R}\right)$$

Now from equation IV

$$\sin\frac{\pi}{m}\frac{E}{Z}=\frac{i}{\frac{Z}{z_1}(1-\cos\theta_0)-\frac{1}{2}\cot\frac{\pi}{m}\sin\theta_0}$$

so that
$$\varepsilon^{\frac{2\pi c}{m}}+\cos\theta_0=\left(\varepsilon^{\frac{2\pi c}{m}}-1\right)\left(1+\frac{e}{Ri}\right)\left\{\frac{Z}{z_1}(1-\cos\theta_0)-\frac{1}{2}\cot\frac{\pi}{m}\sin\theta_0\right.$$

from which $\cos\theta_0$

$$=\frac{\frac{Z}{z_1}\left(\varepsilon^{\frac{2\pi c}{m}}-1\right)\left(1+\frac{e}{Ri}\right)-\varepsilon^{\frac{2\pi c}{m}}-\left[\frac{1}{2}\cot\frac{\pi}{m}\sin\theta_0\left(\varepsilon^{\frac{2\pi c}{m}}-1\right)\left(1+\frac{e}{Ri}\right)\right]}{\frac{Z}{z_1}\left(\varepsilon^{\frac{2\pi c}{m}}-1\right)\left(1+\frac{e}{Ri}\right)+1}.$$

It will be noticed that the second term in the numerato of equation V becomes zero for the special case where $m=$: and it is often neglected for all cases.

To show the application of these formulae to curve plot ting, &c., consider the case of a bi-phase mercury arc havin the following characteristics:

Regd. D.C. E.M.F. (including arc drop), $e=105$ volts
Mean direct current $(\theta=0)i=3{\cdot}25$ amperes

$r_1=1{\cdot}22$ ohms, $x_1=2{\cdot}5$ ohms, $z_1=2{\cdot}78$
$R=1{\cdot}72$ „ , $X=69{\cdot}5$ „ , $Z=69{\cdot}6$

$$c=\frac{R}{X}=0{\cdot}0247$$

The overlap angle θ_0:

$$\cos\theta_0=\frac{\frac{Z}{z_1}(\varepsilon^{\pi c}-1)\left(1+\frac{e}{Ri}\right)-\varepsilon^{\pi c}}{\frac{Z}{z_1}(\varepsilon^{\pi c}-1)\left(1+\frac{e}{Ri}\right)+1}\text{ equation V }(m=2)$$

$$=\frac{(25\times0{\cdot}08\times19{\cdot}8)-1{\cdot}08}{(25\times0{\cdot}08\times19{\cdot}8)+1}=0{\cdot}95$$

$\therefore\ \theta_0=18$ deg.

The maximum value of the line to neutral voltage:

$$E=\frac{z_1 i}{1-\cos\theta_0} \text{ equation IV } (m=2)$$

$$=\frac{2{\cdot}78\times 3{\cdot}25}{0{\cdot}05}=180 \text{ volts}$$

Current curve between 0 and θ_0:

$$i_0=B\varepsilon^{-b\theta}-\frac{2e}{2R-r_1} \text{ equation II } (m=2)$$

$$b=\frac{2R-r_1}{2X-x_1}=\frac{2{\cdot}22}{136{\cdot}5}=0{\cdot}0163.$$

$$B=i+\frac{2e}{2R-r_1}=3{\cdot}25+\frac{210}{2{\cdot}22}=97{\cdot}75$$

For $\theta=0°$ and $18°$, that is, at the commencement and end of the period of commutation during which two arcs burn, the calculations for i_0 are set out in the following table:

θ		$\epsilon^{-b\theta}$	$B\epsilon^{-b\theta}$	$\frac{2e}{2R-r_1}$	i_0
Deg.	Rad.				
0	0	1·0	97·75	94·5	3·25
18	0·314	0·995	97·25	94·5	2·75

Current curve between θ_0 and π:

$$i_1=GE^{-c\theta}-\frac{e}{R}+\frac{E}{Z}\sin(\theta-\gamma) \text{ equation III } (m=2)$$

$$G=\varepsilon^{\pi c}\left(i+\frac{e}{R}-\frac{E}{Z}\sin\gamma\right)$$

$$\gamma=\tan^{-1}\frac{X}{R}=88{\cdot}6 \text{ deg.}$$

$$G=1{\cdot}08\,(3{\cdot}25+61-2{\cdot}59)=66{\cdot}6.$$

Four values of θ have been taken in the interval during which one arc burns alone, and the calculations for i_1 are set out in the following table:

θ Deg.	θ Rad.	$\epsilon^{-c\theta}$	$G\epsilon^{-c\theta}$	$\frac{e}{R}$	$\frac{E}{Z}\sin(\theta-\gamma)$	i_1
18	0·314	0·992	66·1	61	−2·44	2·66
90	1·57	0·960	63·9	61	0·06	2·96
150	2·62	0·937	62·4	61	2·28	3·68
180	3·14	0·925	61·6	61	2·59	3·19

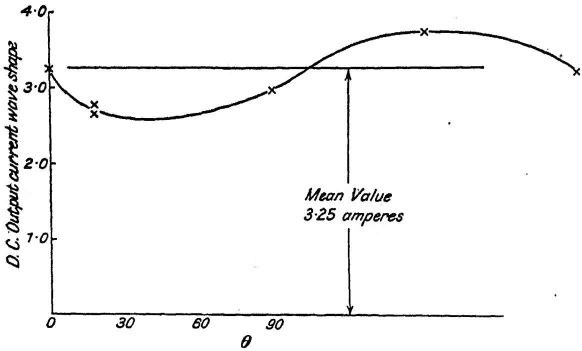


FIG. 49

It will be noticed that there is not complete agreement between either the pairs of current values ($\theta=18$) or ($\theta=0$ and 180), and this is probably due to the fact that neither E nor θ_0 can be evaluated exactly.

Fig. 49 shows the shape of the D.C. wave plotted from the calculated values.

APPENDIX

THE PIRANI GAUGE

IN the bridge circuit shown X is a variable resistance: a, b, g, r, R are fixed resistances, and x, y and z are currents.
Assume $R < X$ so that the current flow is as shown, and that E is constant.

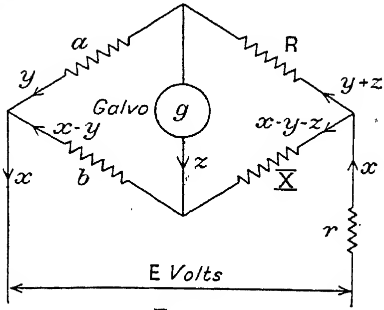


FIG. 50

The following are two P.D. equations for the circuit:

$$ay - b(x-y) - gz = 0$$
$$rx + R(y+z) + ay = E$$

on rearranging

$$-gz + (a+b)y - bx = 0$$
$$Rz + (a+R)y + rx = E$$

by eliminating x and solving for y, it is found that

$$y = \frac{Eb + z(gr - bR)}{r(a+b) + b(a+R)}$$

Now z is a very small quantity, a few microamperes, and if the dimensions of the bridge components are such that $gr \doteqdot bR$

then

$$y = \frac{Eb}{r(a+b) + b(a+R)}$$

similarly

$$x = \frac{E(a+b)}{r(a+b) + b(a+R)}$$

Thus x and y are constant currents and the coil X carries a constant current even though its resistance varies.

A third P.D. equation is

$$gz - X(x-y-z) + R(y+z) = 0$$

so that

$$X = \frac{R(y+z)+gz}{x-y-z}$$

and since R, g, x, and y are constant and z is very small

$$z = KX - k$$

where K and k are constants, so that the galvanometer deflection is proportional to the value of X.

INDEX

Printed in Great Britain by
Butler & Tanner Ltd.,
Frome and London